

基于脉冲控制理论的时滞混沌系统同步技术研究进展

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摘要 基于脉冲控制理论的时滞混沌系统同步的保密通信方案具有更高的保密性、更大的信息处理能力和存储容量, 具有更强的鲁棒性等优点, 应用的前景巨大。深入分析了理想环境和复杂环境下时滞混沌系统脉冲同步控制技术的研究现状, 指出了其目前存在的问题和发展趋势。

关键词 脉冲控制 时滞 混沌系统 同步 趋势

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自从1990年 Pecora 和 Carroll^[1] 首次提出驱动-响应的混沌同步方案并在电子线路设计中实现以来, 混沌同步研究就成为非线性科学中的一个热点, 在保密通信、非线性电路、人体生命科学、激光等众多学科领域取得了初步成果, 展现出了诱人的应用前景^[2-8]。

时滞混沌系统为无穷维状态空间, 能够产生多于维数的正的 Lyapunov 指数, 在避免增加系统维数的同时可以极大地提高混沌系统的复杂性, 是研究的热点^[9-13]。随着混沌同步研究的深入, 各种同步现象不断被发现, 如完全同步^[14]、反同步^[15]、滞后同步^[16]、广义同步^[17]、投影同步^[18]、广义函数投影同步^[19]、相位同步^[20]等。其中, 广义函数投影同步

可以把传输数据从二进制扩展到 M 进制以实现更有效的信息传输, 大大增强了通信的保密性, 引起了人们的研究热情^[21-26]。

混沌同步控制方法主要有滑模变控制^[27-29]、脉冲控制^[30]、状态观测器^[31]、自适应控制^[32]、间歇控制^[33-35]等。1971年, E. Gilbert 和 G. Harasty 首次提出了脉冲控制方法^[36]。20世纪90年代, T. Yang^[37]首次将脉冲控制用于混沌同步控制, 利用很小的控制脉冲间断地传送到接收端以达到驱动-响应系统的同步, 控制能量大大减少, 鲁棒性强, 控制器简单, 被广泛用于混沌同步控制、混杂系统控制、复杂网络控制等诸多环境。

混沌同步控制是混沌保密通信系统的关键理论和技术依托。由于技术上的不足现在所实现的混沌保密通信系统均属于低维的混沌系统, 不一定足够安全。要增加通信安全性, 必须解决两个问题: 使发送信号更加复杂和降低传送信号的冗余度。对于第一个问题, 在不增加系统维数的前提下可以采用时滞混沌系统。对于第二个问题, 脉冲同步是一个很有发展前途的方法。因此, 时滞混沌系统脉冲同步的保密通信方案具有更高的保密性、更大的信息处理能力和存储容量, 具有更强的鲁棒性等优点。

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1 理想环境下基于脉冲控制理论的时滞混沌系统同步

脉冲控制理论首先在连续混沌系统的同步控制中得到了广泛应用^[38-42]。1997 年, Tao Yang^[43]采用比较系统的方法研究了混沌系统的脉冲控制和同步,提出了几条稳定性定理,在混沌系统参数和脉冲控制规律已知的前提下,固定的脉冲间隔的上限被估计出来;把它和混沌掩盖技术相结合设计了一种通信方案,采用 Chua 电路进行了仿真实现。2000 年, X. W. Xie 等人^[44]首次提出了时变脉冲控制技术,与传统固定脉冲控制不同,它的条件更为宽松,脉冲同步间隔也可以进一步增大,脉冲间隔被分为 Δ_{2i} 和 Δ_{2i+1} , $i = 1, 2, \dots$ 两个不同且满足特定比例的脉冲间隔值,此后, Z. G. Li 等^[45,46]和孙继涛等^[47]采用比较系统的方法做了进一步深入研究。文献[48]提出了一种具有适应性的脉冲间隔时变的脉冲同步鲁棒控制模型,并采用连续的 Lorenz 混沌系统进行了仿真实现,研究表明时变脉冲同步控制技术的理论和方法存在进一步扩展的空间,控制条件有继续放宽的可能性。刘光民等^[49]采用 Lyapunov 方法研究了具有通道延时和参数时变的连续混沌系统的脉冲同步。脉冲控制理论在时滞混沌系统同步控制中得到了应用。

不少学者对不存在干扰因素等的理想环境下的同步控制技术进行了研究。2007 年, 樊春霞^[50]采用比较系统方法^[51,52], 在系统矩阵为 Hurwitz 矩阵和正定矩阵时,得到了同步的充分条件,使系统快速地达到 10^{-5} 的量级误差,并在一个多重时滞混沌系统中进行了仿真实现。采用 Lyapunov 方法, 马铁东等^[53]研究了具有时变时滞的混沌系统的脉冲同步,得到一些比较有效和弱保守型的条件。脉冲控制理论在时滞混沌神经网络同步控制中也有一定应用。文献[54—56]中的研究成果仅适用于具有固定时滞的随机神经网络。刘新芝等^[57]和 Li Xiaodi^[58]研究了 Hopfield 神经网络的指数稳定性,得到了新的指数稳定性判据,蒋海军等^[59]研究了具有反应扩散项的时滞神经网络的脉冲控制和广义指数稳定同步,曹进德团队^[60]研究了具有混合时滞的

开关神经网络的同步行为, Song^[61]提出了一些改进的 Lyapunov-Krasovskii 函数,该方法在噪声干扰下是不适用的。以上分析中,脉冲控制器的脉冲同步间隔都是固定的。

2 复杂环境下基于脉冲控制理论的时滞混沌系统同步

实际应用中,外界环境总是很复杂的。内部或信号传输通道噪声干扰、脉冲采样延时、通道传输延时、参数不确定性、参数失配等都或大或小地存在,同步的驱动和响应系统的结构也可能是不同的,它们的参数不确定性也可能是不同的或是时变的,科研人员近几年对复杂环境下的时滞系统脉冲同步控制的鲁棒性进行了分析。

2005 年,杨小帆等^[62]研究了一类时滞混沌系统和神经网络的脉冲控制和系统单个参数存在失配下的准同步行为,同步容许误差是 10^{-4} ,稳定性判据限制过多,不利于实际应用。刘新芝^[63]研究了具有时滞的耦合混沌系统的同步,得到了满足同步阈值的脉冲间隔的上限,可用于具有传输延时的混沌保密通信中。陈武华等^[64]研究了参数时变和范数有界的时滞脉冲系统的鲁棒稳定性, Li Xiaodi^[65]采用比较系统的方法研究了具有无穷或有穷时滞的脉冲函数微分方程的广义指数稳定性,孙继涛等^[66]研究了非线性随机微分时滞系统的稳定性。张卫东等^[67]首次将驱动-响应系统之间的状态变量误差的线性反馈信号作为脉冲控制信号,得到了具有时滞和参数不确定性的不同超混沌系统的脉冲同步渐进稳定条件,提出一个数字通信系统模型,对超陈和超吕混沌系统进行了仿真研究。曹进德等^[68]研究了具有时变时滞和参数失配的两个不同结构的 Lure 混沌系统的广义同步。在时滞混沌神经网络同步控制研究方面,马铁东等^[69]研究了参数不确定的一类时滞神经网络的指数同步鲁棒性,可以用于研究多干扰下的同步。Li Xiaodi 等^[70]研究了具有脉冲和随机干扰的时滞神经网络的指数同步,该方法没有采用复杂的 Lyapunov-Krasovskii 泛函,得到的结果独立于时变时滞。上述研究中,脉冲控制器的脉冲间隔都是固定的。Khadra An-

mar^[71]等研究了一类具有线性时滞的自治脉冲微分系统,分别得到了固定和时变脉冲间隔的系统同步的充分条件,并应用到两个耦合混沌系统同步中。研究结果表明,脉冲间隔越小,具有较大时滞的非线性系统的脉冲同步效果越好,研究中有两个问题有待进一步解决:① 通道噪声和参数失配对同步的影响和② 同步技术如何在保密通信中具体实施。

上述研究中的脉冲同步指的均是完全同步。2009 年,曹进德等^[72]研究了一类时滞混沌系统的投影同步,脉冲间隔是固定的。Zhao Hongyong 等^[73]研究了混沌时滞神经网络的指数反同步,脉冲间隔也是固定的。此外,脉冲控制理论还在分数阶混沌系统^[74—78]、复杂网络^[79—82]等的同步控制中得到应用。王兴元等^[74]研究了分数阶超混沌系统的脉冲同步,研究中把分数阶脉冲系统的稳定性分析简化为整数阶的,并在一个新的超混沌 Chen 系统中进行了仿真实现。郑松^[80]把自适应控制和脉冲控制技术相结合,研究了具有时滞的时变耦合复杂动力学网络的广义同步。

脉冲同步可以与常规的加密方法结合,增加信息传输的安全性。混沌加密方法中,混沌掩盖适用于传输模拟信号,混沌调制、混沌切换和混沌扩频适于传输数字信号。混沌调制是将信号隐藏在系统参数里,其保密性能较好,然而它也会受到噪声干扰、信号衰减以及信道带宽等问题带来的影响,鉴于脉冲控制技术鲁棒性强等特点,将时滞混沌系统的脉冲同步与混沌调制相结合,可以在一定程度上缓解其不足,提高系统的安全性。

3 结论

纵观国内外研究现状,可以得到以下几个结论:①脉冲控制理论正被应用到生物、物理、医学等许多领域,用于解决该领域的连续混沌系统、时滞混沌系统、分数阶混沌系统、复杂网络的控制同步;②脉冲控制理论在时滞混沌系统的同步控制研究中已经取得了一些成果,但是还有不少问题有待进一步探索;③时滞混沌系统脉冲同步研究的主要是完全同步,反同步、投影同步各有一篇论文,关于广义函数投影同步的研究很少;④复杂环境下时滞混

沌系统的脉冲同步正在引起越来越多研究人员的关注,对通道传输延时、噪声干扰、参数失配等问题的研究一方面可以完善丰富时滞混沌系统脉冲控制同步的理论研究,提高同步控制的鲁棒性,另一方面研究成果可以直接用于现场,加快了理论成果向实际应用的转化过程;⑤时变脉冲同步控制方法在连续混沌系统的控制中被重点研究,在时滞混沌系统中研究较少,可以考虑把它应用到时滞系统中,以得到更加宽松和实用的同步控制条件;⑥脉冲同步可以与混沌调制相结合,混沌源选择时滞混沌系统,可以进一步提高混沌保密通信的安全性和鲁棒性。

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Recent Advances of Synchronization of a Class of Time Delayed Chaotic Systems Based on Impulsive Control Theory

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[Abstract] The secure communication scheme of synchronization of a class of time delayed chaotic systems based on impulsive control theory is more secure. And it has greater information processing power, storage capacity, stronger robustness and tremendous application prospects, and other advantages. The present situation of impulsive synchronization control schemes of time delayed chaotic systems under the ideal environment and complex environment are analyzed. Existing problems and development trends of the study are pointed out.

[Key words] impulsive control time delayed chaotic systems synchronization trend