

参数经局部扰动的分数阶混沌 Chen 系统的自适应追踪控制与同步

彭艳艳 李庶民 何书霞

(昆明理工大学理学院数学系, 昆明 650093)

摘要 针对 Chen 混沌系统, 研究了参数经局部扰动的混沌系统的自适应同步。基于分数阶系统的稳定定理和自适应反馈控制, 设计了自适应反馈控制器和未知参数的辨识规则, 实现了参数经局部扰动的分数阶 Chen 混沌系统同给定信号的同步。数值仿真证实了所设计的控制器及未知参数的辨识规则的有效性。

关键词 自适应反馈控制 混沌同步 自适应同步

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Fractional calculus, as a branch of mathematical analysis, deals with the generalization of the concepts of differentiation and integration to arbitrary non-integer orders. Although the fractional calculus has a long history, its applications to physics and engineering have just started in the recent decades^[1]. Many systems are known to display fractional order dynamics, such as viscoelastic system^[2], trode-electrolyte polarization^[3] and electromagnetic wave^[4]. Beside the modelling of dynamical systems by using the means of the fractional calculus, applications of this mathematical branch have been reported in many engineering areas such as image processing^[5], electromagnetism^[6], mechanics^[7] and bioengineering^[8].

Recently, synchronization of chaotic fractional-order systems has become more and more interesting to researchers due to its potential applications in secure communication and control processing. For example, in reference [9] chaos synchronization of two fractional Lu systems has been studied. Also, synchronization of

two chaotic fractional Chen systems and synchronization of two chaotic fractional Chua systems have been presented in reference [10], respectively.

To our best knowledge, most of the methods mentioned above and many other existing synchronization methods mainly have ignored the uncertainties. Since noise is ubiquitous in both nature and man-made systems, synchronization of concrete models is unavoidably subject to internal and external noise. Here, additional partly perturbed parameters in dynamic equations of slave systems are considered. Based on the ideas of feedback control and the stability theorem of fractional-order systems, the synchronization of two mismatched energy resource systems is addressed by employing adaptive feedback control.

1 System description

Chaotic Chen system is described as^[11]

$$\begin{cases} \frac{dx}{dt} = a(y - x) \\ \frac{dy}{dt} = (c - a)x - xz + cy \\ \frac{dz}{dt} = xy - bz \end{cases} \quad (1)$$

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第一作者简介: 彭艳艳(1987—), 山西运城人, 硕士研究生, 研究方向: 微分方程理论与动力系统。

Where x , y and z are state variables, a , b and c are the related parameters. The Chen system exhibits a chaotic behavior at the parameters $a = 35$, $b = 3$ and $c = 28$. Its fractional version is in the form^[12],

$$\begin{cases} \frac{d q_1}{d t} x = a(y - x) \\ \frac{d q_2}{d t} y = (c - a)x - xz + cy \\ \frac{d q_3}{d t} z = xy - bz \end{cases} \quad (2)$$

2 Synchronization between one fractional-order Chen's systems with partly perturbed parameters and the master system

Here, chaos synchronization issue between two chaotic fractional-order Chen systems via master-slave configuration with the state feedback scheme are investigated. The familiar method for constructing the master-slave configuration is applied which was introduced by Carroll and Pecora^[13]. Respectively, consider the master and slave systems as the following:

$$\begin{cases} \frac{d q_1}{d t} x_m = a(y_m - x_m) \\ \frac{d q_2}{d t} y_m = (c - a)x_m - x_m z_m + cy_m \\ \frac{d q_3}{d t} z_m = x_m y_m - bz_m \end{cases} \quad (3)$$

and

$$\begin{cases} \frac{d q_1}{d t} x_s = ay_s - (a + d_1(t))x_s + u_1 \\ \frac{d q_2}{d t} y_s = (c - a)x_s - x_m z_s + (c + d_2(t))y_s + u_2 \\ \frac{d q_3}{d t} z_s = x_m y_s - (b + d_3(t))z_s + u_3 \end{cases} \quad (4)$$

Where $d_i(t)$ ($i = 1, 2, 3$) are unknown parameter disturbances of the slave system which needs to be estimated, and u_1, u_2, u_3 are linear control functions to be designed.

Set

$$e_1 = x_s - x_m, \quad e_2 = y_s - y_m, \quad e_3 = z_s - z_m, \quad (5)$$

Then one obtains the error system:

$$\begin{cases} \frac{d q_1}{d t} e_1 = a(e_2 - e_1) - d_1(t)x_s + u_1 \\ \frac{d q_2}{d t} e_2 = (c - a)e_1 - x_m e_3 + ce_2 + d_2(t)y_s + u_2 \\ \frac{d q_3}{d t} e_3 = x_m e_2 - be_3 - d_3(t)z_s + u_3 \end{cases} \quad (6)$$

The goal of the control is to find an effective controller $u = (u_1, u_2, u_3)$ with a parameter estimation update law such that the slave system (4) asymptotically synchronizes the master system (3), i. e., $\lim_{t \rightarrow \infty} e(t) = 0$, where $e(t) = (e_1, e_2, e_3)^T$.

For two systems (3) and (4) without controls $u = (u_1, u_2, u_3)$, if the initial condition $(x_m(0), y_m(0), z_m(0)) \neq (x_s(0), y_s(0), z_s(0))$ then the trajectories of two systems will quickly separate from each other and become irrelevant. However, when controls are applied, the two systems will approach synchronization for any initial conditions through appropriate control functions. The following theorem shows that chaotic systems (3) and (4) can be synchronized effectively by the following proposed adaptive feedback controller.

Theorem Chaotic systems (3) and (4) can be synchronized globally and asymptotically for any different initial condition with the following feedback adaptive controller:

$$\begin{cases} u_1 = -ce_2, \\ u_2 = -ke_2, \\ u_3 = -ue_3, \end{cases} \quad (7)$$

and the following parameter estimation adaptive update law:

$$\begin{cases} \frac{d q_1}{d t} d_1(t) = e_1 x_s, \\ \frac{d q_2}{d t} d_2(t) = -e_2 y_s, \\ \frac{d q_3}{d t} d_3(t) = e_3 z_s, \end{cases} \quad (8)$$

where $c=28, k>28, u>0$, and $(0, 0, 0)$ are the estimations of unknown parameter disturbances $(d_1(t), d_2(t), d_3(t))$, respectively.

Proof Substituting formula (7) into formula (6) leads to the following error system:

$$\begin{cases} \frac{d q_1}{d t} e_1 = a(e_2 - e_1) - d_1(t) x_s - c e_2, \\ \frac{d q_2}{d t} e_2 = (c - a) e_1 - x_m e_3 + c e_2 + d_2(t) y_s - k e_2, \\ \frac{d q_3}{d t} e_3 = x_m e_2 - b e_3 - d_3(t) z_s - u e_3, \end{cases} \quad (9)$$

Where $e_{d_1} = d_1(t), e_{d_2} = d_2(t), e_{d_3} = d_3(t)$. Based on the idea of the stability theory of fractional-order systems^[14], consider the following function:

$$\begin{aligned} J &= [e_1, e_2, e_3, e_{d_1}, e_{d_2}, e_{d_3}] \times \\ &\left(\frac{d q_1}{d t} e_1, \frac{d q_2}{d t} e_2, \frac{d q_3}{d t} e_3, \frac{d q_1}{d t} e_{d_1}, \frac{d q_2}{d t} e_{d_2}, \frac{d q_3}{d t} e_{d_3} \right) = \\ &e_1 \frac{d q_1}{d t} e_1 + e_2 \frac{d q_2}{d t} e_2 + e_3 \frac{d q_3}{d t} e_3 + e_{d_1} \frac{d q_1}{d t} e_{d_1} + \\ &e_{d_2} \frac{d q_2}{d t} e_{d_2} + e_{d_3} \frac{d q_3}{d t} e_{d_3} = e_1 (a(e_2 - e_1) - d_1(t) x_s - \\ &c e_2) + e_2 ((c - a) e_1 - x_m e_3 + c e_2 + d_2(t) y_s - k e_2) + \\ &e_3 (x_m e_2 - b e_3 - d_3(t) z_s - u e_3) + e_{d_1} (e_1 x_s) + e_{d_2} (-e_2 y_s) + \\ &e_{d_3} (e_3 z_s) = -a e_1^2 - (k - c) e_2^2 - (b + u) e_3^2 = -e^T P e, \end{aligned}$$

Where, $e = (e_1, e_2, e_3)^T, P = \text{diag}(p_1, p_2, p_3)$ ($p_1 = a > 0, p_2 = (k - c) > 0$, and $p_3 = (b + u) > 0$). It is obvious that, for suitable values of p_1, p_2, p_3 , the matrix P is positive definite, and J is negative semidefinite. So $J \leq 0$. According to the idea of the stability theory of fractional-order systems^[14], one obtains $e_1, e_2, e_3 \rightarrow 0$ as $t \rightarrow \infty$, namely $\lim_{t \rightarrow \infty} \|e(t)\| = 0$. So, the synchronization between slave (4) and master system (3) can be achieved. This completes the proof.

3 Simulation results

The initial states of the master and slave systems are taken as

$(x_m(0), y_m(0), z_m(0)) = (6, 2, -8)^T$ and $(x_s(0), y_s(0), z_s(0)) = (0.1, 1, 0.1)^T$, respectively. For convenience, we always let $(a, b, c) = (35, 3, 28)$ and choose $q = (0.975, 0.975, 0.975)$. The time step-length is 0.008, and the first 1000 points are removed. fig. 1 and fig. 2 shows the effectiveness of the proposed control scheme, the synchronization is achieved.

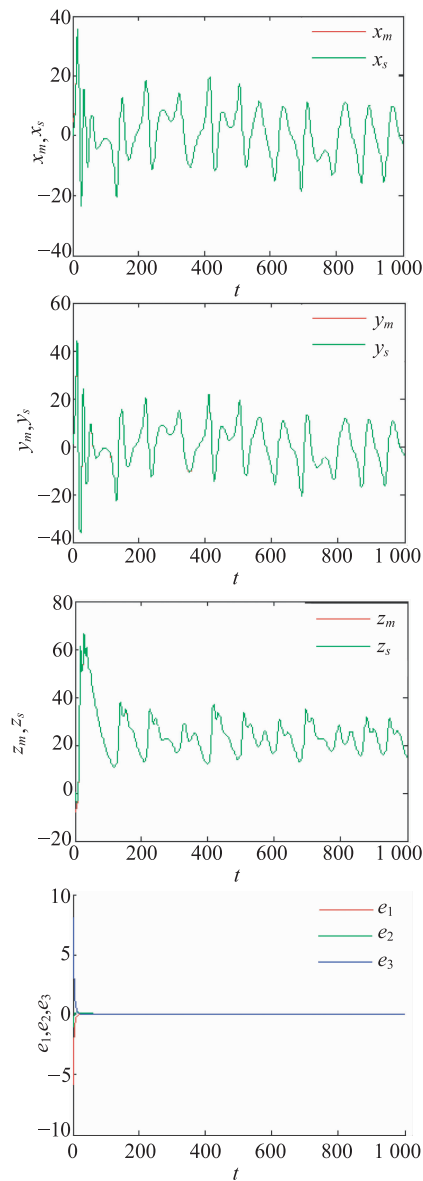


Fig.1 Chaos synchronization between eqs (3) and (4); Here, $k=29, u=30$

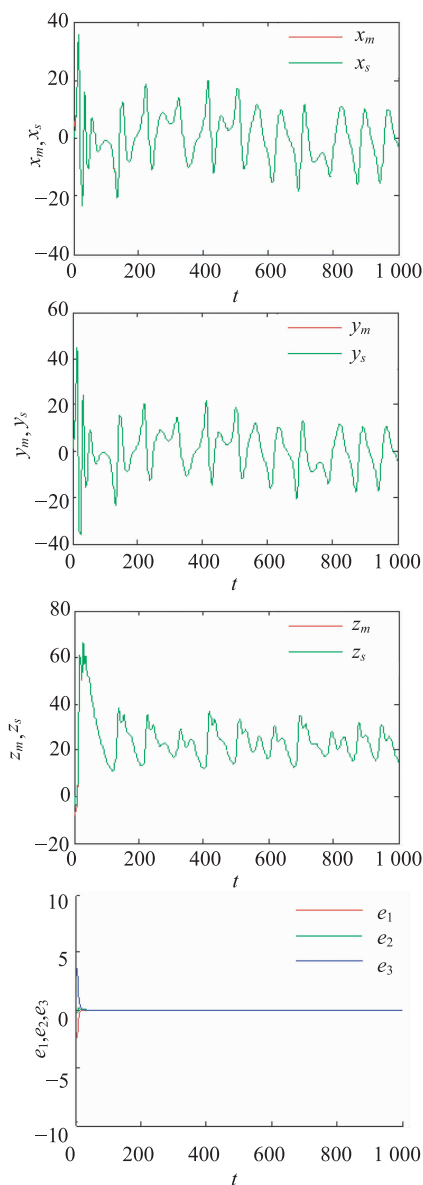


Fig.2 Chaos synchronization between eqs (3) and (4); Here, $k = 29$, $u = 35$

4 Conclusions

The adaptive synchronization between one fractional-order Chen's systems with partly perturbed parameters and the drive system are investigated. Based

on the ideas of adaptive feedback control and stability theory of fractional-order systems, a linear adaptive feedback synchronization controllers was proposed. The proposed controller synchronizd the fractional-order Chen's systems with partly perturbed parameters and the drive system. All the theoretical results are verified by numerical simulations to demonstrate the effectiveness of the proposed synchronization schemes.

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The Geom/Geom/1 Queue with Delayed Vacation and Vacation Interruptions

SHI Hai-yan¹, WEI Chun²

(Capital Normal University¹, Beijing 100048, P. R. China; Shanxi Datong University², Datong 037000, P. R. China)

[Abstract] A Geom/Geom/1 queue with delayed vacation and vacation interruptions is studied. Even if the server has completed all customers, it does not immediately enter working vacation. Only when there are no customers during the delayed vacation, the server begins a working vacation. Vacation interruption policy during the working vacation is also introduced. Using quasi-birth-and-death process and matrix-geometric solution method, the steady-state distributions for queue length and the probability generating function are given. Then the stochastic decomposition structure of queue length and sojourn time in the stationary state is gained and obtained the distributions for additional queue length and additional delay time.

[Key words] delayed vacation vacation interruption matrix-geometric solution stochastic decomposition

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Adaptive Synchronization of Perturbed Chen's Fractional-order Chaotic Systems *via* Adaptive Feedback Control

PENG Yan-yan, LI Shu-min, HE Shu-xia

(Department of Applied Mathematics, Kunming University of Science and Technology, Kunming 650093, P. R. China)

[Abstract] The adaptive synchronization between one fractional-order Chen's systems with partly perturbed parameters and the drive system are investigated. Based on the ideas of feedback control and stability theory of fractional-order systems, synchronization between these two chaotic systems is achieved by proposing a new adaptive feedback controller and a parameter estimation update law. Numerical simulations are presented to demonstrate the analytical results.

[Key words] adaptive feedback control chaos synchronization adaptive synchronization