

# 基于 $H_\infty$ 反馈控制的网络拥塞流速控制器设计

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**摘 要** 为了解决现代高速通信网络拥塞控制问题,采用频域设计方法,将不确定时滞系统转化为带有未建模动态边界的多不确定系统;根据鲁棒镇定及系统性能指标要求,将流量控制的网络拥塞控制设计问题转化为共同的工程混合敏感的应用问题,然后分析设计出了理想的  $H_\infty$  控制器。证明采用频域设计方法的拥塞控制的  $H_\infty$  反馈控制器,可以实现防止拥挤和提高网络应用效率的目标。实例已表明,该方法简单易用。

**关键词** 流速  $H_\infty$  反馈控制 网络拥塞 鲁棒镇定

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There are two congestion control methods in relatively common use at the present. One is the control based on flow rate, in which an original sends data package at a given flow rate, then a data package-occurred rate is accommodated by the feedback information from network. The other is the window control, in which a terminal commands the original send data at a given wide window, the width is adjusted by the feedback information. The congestion control method based on flow rate, owing to being simple and easy to realize, is being more and more prevalently applied in high speed network such as ATM *etc.*, and has arisen many researchers' interest. Moreover, an important factor that has to be considered is the problem on delay and multi-delays in the design of the feedback controller based on flow rate for network congestion control, which has been discussed in several papers recent years. However one of the mostly used methods is  $H_\infty$  robust control. For example, a flow rate controller which is designed based on  $H_\infty$  theory is used to solve

the problem on a multi-delays with time-varying uncertainty in multi-original single-bottleneck network<sup>[1]</sup>. The output flow rate information in bottleneck is made use to improve  $H_\infty$  feedback controller which is designed by making use of the error information of expected queue length. The improvement has enhanced the running speed and minished the tracking error<sup>[2]</sup>. Based on the above literatures, this paper will propose a simple and applicable method, to design a robust  $H_\infty$  congestion feedback controller of the multi-original single-bottleneck network, to prevent congestion, maximize the efficiency of the network, and also to realize robust stabilization so that time-delay influence will be demolished.

## 1 Problem Description

A multi-original single-bottleneck network congestion feedback control system is shown in fig. 1.

Where,  $q(t) \geq 0$  denotes the actual length of the data buffer queue of bottleneck node,  $q_e(t) > 0$  is the expected maximum length of the data buffer queue, is the data output rate adjusted by congestion control feedback system from each original;  $r_i(t - \tau_i)$  is the data input rate of each original on bottleneck node;  $\tau_i$

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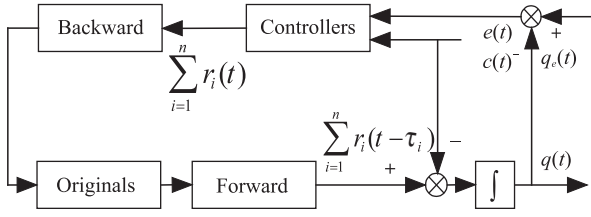


Fig. 1 A network congestion control feedback system

stands for the uncertain time-delay of each original, satisfying  $0 \leq \tau_i(t) \leq \tau_m$ ;  $c(t)$  is the data output rate of bottleneck node. The dynamic model of this system can be expressed as<sup>[3]</sup>:

$$\dot{p}(t) = \sum_{i=1}^n r_i(t - \tau_i) - c(t) \quad (1)$$

**Lemma 1**<sup>[4]</sup>: Given plant  $P(s)$ , controller  $K(s)$ , weighted function  $W_q(s)$  with additive uncertainty,  $P = P_0(1 + W_q)$ , norm uncertainty,  $\Delta(s), \bar{\Delta}(s) \in \bar{B}\bar{H}_\infty$ .

(1) For additive uncertainty of any plant, the necessary and sufficient condition for system's robust stabilization is that: ① there is a controller  $K$  which makes the feedback control system shown in fig. 2 be stable for any  $\Delta(s) \in \bar{B}\bar{H}_\infty$ .

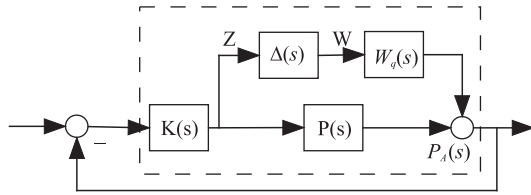


Fig. 2 Control system with additive uncertainty

②  $\|(1 + KP_0)^{-1}KW_q\|_\infty < 1$ , i. e.,  $(I + KP_0)^{-1}KW_q \in \bar{B}\bar{H}_\infty$ ;

(2) For multiplicative uncertainty, the necessary and sufficient condition for system's robust stabilization is that: ① there is a controller  $K$  which makes the feedback control system shown in fig. 3 be stable for any  $\Delta(s) \in \bar{B}\bar{H}_\infty$ .

②  $\|(I + P_0K)^{-1}PKW_q\|_\infty < 1$ , i. e.,  $(I + P_0K)^{-1}PKW_q \in \bar{B}\bar{H}_\infty$ .

**Lemma 2**: Let  $P = N_1D_1^{-1} = N_2D_2^{-1} \in RL_\infty$  and

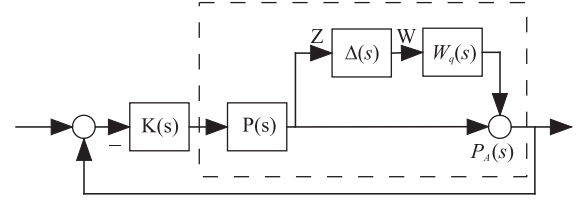


Fig. 3 Control system with multiplicative uncertainty

$\begin{bmatrix} N_2 \\ D_2 \end{bmatrix} = \begin{bmatrix} N_1 \\ D_1 \end{bmatrix} W$ , further more, if  $P = ND^{-1} \in RH_\infty$  and right relatively prime factorable, then  $D^{-1} \in RH_\infty$ , and can take  $W = D^{-1}$ .

Therefore, the above feedback system's  $P_0$  can be relatively prime factored as  $P_0 = ND^{-1}$ , the set for  $K$  to stabilize  $P_0$  is  $\left\{ \frac{U + DW}{V - NW} : NU + DV = 1 \right\}$ , where  $U, V, W$  are all stable, regular, real rational function.

## 2 The $H_\infty$ Feedback Controller Design for Congestion Control

Considering the principia of every original fairness (set the number of originals is  $n$ ), set  $r_i(t)$  is determined by following control law:

$$r_i(t) = k_{ei}e(t) + \frac{1}{n}k_{ci}c(t) \quad (2)$$

where  $e(t) = qe(t) - q(t)$ , then a feedback system block is shown as fig. 4<sup>[5]</sup>.

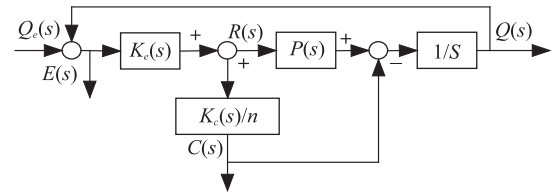


Fig. 4 Block chart of feedback system

In fig. 4,  $P(s)$  stands for the delay point, is a multi-input single-output system (MISO), its transfer function is  $P(s) = [e^{-\tau_1 s}, \dots, e^{-\tau_n s}]$ ;  $K_e(s)$  and  $K_c(s)$  stand for the feedback controllers, there are single-input multi-output systems, their transfer functions are

$$K_e(s) = [K_{e1}^T(s), \dots, K_{en}^T(s)]^T.$$

and

$$K_c(s) = [K_{c1}^T(s), \dots, K_{cn}^T(s)]^T$$

respectively;  $R(s) = [R_1^T(s), \dots, R_n^T(s)]^T$  is the controlled output rate of originals.

$$\text{Suppose } G(s) = \frac{P(s)}{s} = \frac{1}{s} [e^{-\tau_1 s}, \dots, e^{-\tau_n s}], G_0$$

$(s) = \frac{1}{s} [1, \dots, 1]$  then  $\left| \frac{G(s)}{G_0(s)} - 1 \right| [ |e^{-\tau_1 s} - 1|, \dots, |e^{-\tau_n s} - 1| ] \leq |W_T(j\omega)|$ , where  $W_i(j\omega) = [W_{i1}(j\omega), \dots, W_{im}(j\omega)]$ , and for  $\forall \omega \in R, 0 \leq \tau_i(t) \leq \tau_m$  we have

$$|W_n(j\omega)| \geq |e^{-j\tau_m \omega}| \quad (3)$$

**Theorem 1** If the network-congestion-considered uncertain delay system is enabled robust stabilization, at the same time, the network reaches its maximum utilizing efficiency, and controller  $K_e(s)$  appears 0 polar, then, performance index  $\left\| \gamma^{-1} W_s D(V - NW) \right\|_{\infty} \leq 1$  must be satisfied. Where,  $N(s), D(s)$  are the relatively prime factorization of  $G_0(s)$ , i. e.,  $N(s)D^{-1}(s) = G_0(s)$ , and  $D(s) = \frac{s}{s + \alpha}, N(s) = \frac{1}{s + \alpha} [1, \dots, 1]$ ,  $\alpha$  is any scalar larger than 0;  $U(s) = \frac{\alpha}{n} [1, \dots, 1]^T, V(s) = 1, N(s)U(s) + D(s)V(s) = 1$ ;  $W(s)$  is a stable, regular and real rational function;  $W_s(s)$  is the sensitive weighted function.

**Proof:** Being perceived, the theorem is involved with an optimal problem on mixed sensitivity familiar to engineering applications.

From lemma 1, the above mentioned uncertain delay system can be robust stabilized, if and only if the nominal system of  $G_0(s)$  can be stabilized, and satisfies performance index<sup>[6]</sup>:

$$\|W_i(s) G_0(s) K_e(s) (1 + G_0(s) K_e(s))^{-1}\|_{\infty} \leq 1 \quad (4)$$

From lemma 2, the nominal system can be robust

stabilized, if and only if it satisfies

$$K_e(s) = (U(s) + D(s)W(s)(V(s) - N(s)W(s)))^{-1} \quad (5)$$

Furthermore derivative to  $e(t)$ , we get

$$\dot{e}(t) = - \sum_{i=1}^n K_{ei} e(t - \tau_i) - \frac{1}{n} \sum_{i=1}^n K_{ci} c(t - \tau_i) + c(t) \quad (6)$$

That is

$$\frac{E(s)}{C(s)} = \frac{1 - \frac{1}{n} \sum_{i=1}^n K_{ci}(s) e^{-\tau_i s}}{s + \sum_{i=1}^n K_{ei}(s) e^{-\tau_i s}} \quad (7)$$

For guaranteeing that the steady error of  $q(t)$  tracking  $q_e(t)$  is 0, from above formula, we have  $\sum_{i=1}^n K_{ci}(0) = n$  and  $\sum_{i=1}^n K_{ei}(0) \rightarrow \infty$ ; in view of the every original fairness, take  $K_{ci}(0) = 1$ ; and knowing that  $K_{ei}(s)$  has a polar  $s=0$ , therefore from eqn.

(5) we know  $V(0) = N(0)W(0)$ , i. e.,  $W(0) = \frac{\alpha}{n} [1, \dots, 1]^T$ ; set  $W(s) = \frac{\alpha}{n} [1, \dots, 1]^T F(s)$ , then there is  $F(0) = 1$  obviously, thus

$$K_e(s) = \frac{\alpha}{n} \left( \frac{1 + \frac{sF(s)}{s + \alpha}}{1 - \frac{\alpha F(s)}{s + \alpha}} \right) [1, \dots, 1]^T \quad (8)$$

From fig. 4, we can know

$$E(s) = q_e((1 + G_0(s)K_e(s))^{-1} + \left( \frac{1}{s} - \frac{1}{n} G_0(s)K_c(s)C(s)(1 + G_0(s)K_e(s)) \right)^{-1}) \quad (9)$$

In order to make the network attain a maximum efficiency,  $\|E(s)\|_{\infty}$  should be small as soon as possible, can let  $\frac{1}{s} - \frac{1}{n} G_0(s)K_c(s) = 0$ , i. e.,  $\sum_{i=1}^n K_{ci}(s) = n$ . From the every original fairness, can take  $K_{ci}(s) = 1$ . Therefore, for guaranteeing net-utilizing efficiency, following  $H_{\infty}$  performance index needs to satisfy with<sup>[7]</sup>:

$$\|\gamma^{-1} W_s(s) (1 + G_0(s)K_e(s))^{-1}\|_{\infty} \leq 1 \quad (10)$$

Where  $\gamma > 0$  is the  $H_\infty$  performance index chosen.

In order for making controller  $K_e(s)$  present polar 0, and in the mean time, for guaranteeing that  $E(s)$  has a larger attenuation in low frequency band, take  $W_s(s) = \frac{1}{s^2}$ . Synthesizing performance indices eguas. (4)

and (10), we have

$$\left\| \frac{W_t(s) G_0(s) K_e(s) (1 + G_0(s) K_e(s))^{-1}}{\gamma^{-1} W_s(s) (1 + G_0(s) K_e(s))^{-1}} \right\|_\infty \leq 1 \quad (11)$$

Making use of egua. (5), we get

$$\left\| \frac{\gamma^{-1} W_s(s) D(s) (V(s) - N(s) W(s))}{W_t(s) N(s) (U(s) + D(s) W(s))} \right\|_\infty \quad (12)$$

The theorem is proved.

Note: Formular egua. (12) can be further transformed as

$$\left\| \frac{W_t(s) \frac{\alpha}{s + \alpha} \left( 1 + \frac{s}{s + \alpha} F(s) \right)}{\gamma^{-1} W_s(s) \frac{s}{s + \alpha} \left( 1 + \frac{\alpha}{s + \alpha} F(s) \right)} \right\|_\infty \leq 1 \quad (13)$$

Adopting the frequency domain training method, we can yield  $F(s)$  from egua. (13); thereby get a congestion-controlling  $H_\infty$  feedback controller.

### 3 Computing Example

Set the bottleneck node output rate of a congestion-controlling system be  $c(t) = 1\,000 + 100\sin(0.1t)$ ,  $t \geq 0$ ; the expected length of buffer queue is  $q_e(t) = 100$ ; the maximum delay of the system is  $\tau_m = 0.1$ ;  $H_\infty$  performance index is took as  $\gamma = 1$ .

Then, it is known that  $|e^{-j\tau_m\omega} - 1| \leq \left| \frac{0.21j\omega}{0.1j\omega + 1} \right|$ ,  $\forall \omega \in R$ . Thus can choose  $W_t(s) = \left| \frac{0.21s}{0.1s + 1} \right|$ , and choose weighted sensitivity function  $W_s(s) = \frac{1}{s^2}$ ,  $G_0(s) = \frac{1}{s} [1, \dots, 1]$ . From the every original fairness, only need to research one of those originals'  $H_\infty$  controllers  $K_{ei}(s)$ . Thus yield:

$$K_{ei}(s) = \frac{1}{s} \frac{s^4 - 1}{s^4} \frac{-\psi(s)}{1 + \psi(s)}.$$

Where,

$$\psi(s) = \frac{s^2(0.1s + 1)(s + 2.11)}{(0.21s^3 + 0.7s^2 + 1.17s + 1)(s - 2.11)}.$$

### 4 Conclusion

The design problem on the robust  $H_\infty$  feedback controller of the congestion-controlling for multi-original single-bottleneck network has been studied in this paper. A network-congestion-control system has been dynamically modeled, then the congestion-controlling  $H_\infty$  feedback controller has been designed, and the performance requirement to the controller has been analyzed. The paper has proved that the  $H_\infty$  feedback controller of congestion control get the goal of preventing congestion and maximizing the efficiency of network application by adopting frequency domain design method. The method can realize avoiding network-congestion and maximizing the network-utilizing efficiency. From the computing process in the example demonstrated, the method proposed has shown its simplicity and applicability.

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## Strengthen Severe Corrosion Structure by CFRP-bonded

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[**Abstract**] A salt factory workshop was used in severe corrosion environment for ages, structure damaged seriously, was unsafe very much, must be reinforced. Based on analysis and demonstration, determined that to reinforce and antisepsis the workshop by comprehensive measures of mainly CFRP-bonded. The principal points of design and construction are introduced. After construction, the workshop has been in operation more than 4 years, and it shows that reinforcing effect is good.

[**Key words**] severe corrosion strengthen CFRP-bonded patch antisepsis

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## A Flow Rate Controller Design for Network Congestion Based on $H_\infty$ Feedback Control

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[**Abstract**] For solving a problem on network congestion control in modern high-speed communication networks, frequency domain design method is adopted, to transform uncertain time-delay system into multiplicative uncertainty system with unmodeled dynamic boundary. According to the requirement of the robust stabilization and performance index of system, the feedback controller design problem on robust  $H_\infty$  congestion control of the high-speed communication networks which based on flow rate control was converted into the common engineering application problem on mixed-sensitivity, then worked out the desirable  $H_\infty$  controller by the analytic method. The  $H_\infty$  feedback controller of congestion control can get the goals of preventing congestion and maximizing the efficiency of network application by adopting frequency domain design method. The method is demonstrated be simple in use and applicable by an example.

[**Key words**] flow rate  $H_\infty$  feedback controller network congestion robust stabilization