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具有脉冲的二阶 Hamilton 系统的周期解

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摘 要 在不假定强制性的条件下,利用最小作用原理建立了具有导数脉冲的二阶非自治哈密顿系统周期解的一个新的存在性定理,改进了相关结果。

关键词 周期解 脉冲 最小作用原理

中图法分类号 O175.8; **文献标志码** A

Consider the non-autonomous second order Hamiltonian systems

$$\begin{cases} \ddot{u}(t) = \nabla F(t,u(t)), \quad \text{a. e. } t \in [0,T] \\ \dot{u}(0) - \dot{u}(T) = 0 \\ u(0) - u(T) = 0 \end{cases} \tag{1}$$

with the impulsive conditions

$$\Delta \dot{u}_i(t_j) := \dot{u}_i(t_j^+) - \dot{u}_i(t_j^-) = I_{ij}(u_i(t_j^-)),$$
$$i = 1,2,\cdots,N; j = 1,2,\cdots,p, \tag{2}$$

where N, p are fixed positive integers, $T > 0$ and $F:[0,T] \times \mathbb{R}^N \to \mathbb{R}$ is T- periodic in its first variable, $I_{ij}: \mathbb{R} \to \mathbb{R}$, $i = 1,2,\cdots,N$, $j = 1,2,\cdots,p$, are continuous, and satisfy the following assumptions:

(A) $F(t,x)$ is measurable in t for all $x \in \mathbb{R}^N$ and continuously differentiable in x for a. e. $t \in [0,T]$, and there exist

$a \in C(\mathbb{R}^+,\mathbb{R}^+), b \in L^1([0,T];\mathbb{R}^+)$,

such that

$|F(t,x)| \leqslant a(|x|)b(t)$, $|\nabla F(t,x)| \leqslant a(|x|)b(t)$,

for all $x \in \mathbb{R}^N$ and a. e. $t \in [0,T]$.

(B) There exist constants $a_{ij} \geqslant 0, b_{ij} \geqslant 0, \gamma_{ij} \in [0,1)$, $i = 1,2,\cdots,N$, $j = 1,2,\cdots,p$, such that

$$|I_{ij}(s)| \leqslant a_{ij} + b_{ij}|s|^{\gamma_{ij}},$$

for every $s \in \mathbb{R}$, $i = 1,2,\cdots,N$, $j = 1,2,\cdots,p$.

The corresponding function φ on H_T^1 is given by

$$\varphi(u) = \frac{1}{2}\int_0^T |\dot{u}(t)|^2 \mathrm{d}t + \int_0^T F(t,u(t))\mathrm{d}t + \sum_{i=1}^N \sum_{j=1}^p \int_0^{u(t_j)} I_{ij}(s)\mathrm{d}s,$$

for $u \in H_T^1$, where

$H_T^1 = \{u:[0,T] \to \mathbb{R}^N \mid u$ is absolutely continuous, $u(0) = u(T)$ and $\dot{u} \in L^2([0,T];\mathbb{R}^N)\}$ is a Hilbert space with the norm defined by

$$\|u\| = \left[\int_0^T |u(t)|^2 \mathrm{d}t + \int_0^T |\dot{u}(t)|^2 \mathrm{d}t\right]^{\frac{1}{2}},$$

for each $u \in H_T^1$.

In a similar way to Proposition 4.1 in Nieto and O Regan[1], we can prove that $\varphi(u)$ is continuously differentiable, weakly lower semi-continuous on H_T^1 and

$$(\varphi'(u),v) = \int_0^T (\dot{u}(t),\dot{v}(t))\mathrm{d}t + \int_0^T (\nabla F(t,u(t)), v(t))\mathrm{d}t + \sum_{i=1}^N \sum_{j=1}^p I_{ij}(u_i(t_j))v_i(t_j),$$

for $u,v \in H_T^1$. Moreover, the weak solutions of problem (1)-(2) correspond to the critical points of φ.

Recently, [1] studied the existence of solutions of second order differential equations with impulses via variational method. However, they only considered one single equation with impulses. In this paper, we investigate the impulsive problem equas. (1)—(2) by using the least action principle.

1 Main Results

Theorem 1 In addition to assumption (B), sup-

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pose further that $F(t,x) = F_1(t,x) + F_2(x)$, where F_1 and F_2 satisfy assumption (A), (B) and the following conditions:

(i) there exist $g, h \in L^1([0,T];\mathbb{R}^+)$ and $\gamma \in [0,1)$ such that

$$|\nabla F_1(t,x)| \leqslant g(t)|x|^{\gamma} + h(t),$$

for all $x \in \mathbb{R}^N$ and a.e. $t \in [0,T]$;

(ii) there exist constants $0 \leqslant r_1 < \frac{2\pi^2}{T^2}, r_2 \in [0,+\infty)$ such that

$$|\nabla F_2(x) - \nabla F_2(y)| \leqslant r_1|x-y| + r_2,$$

for all $x, y \in \mathbb{R}^N$;

(iii) there exist constant $k \in [1,+\infty)$ such that

$$\inf_{|x|\geqslant k}\Big[\frac{1}{|x|^{2\gamma}}\int_0^T F(t,x)\,dt - \sum_{i=1}^N\sum_{j=1}^p b_{ij}2^{\gamma_{ij}+1}|x|^{\gamma_{ij}+1-2\gamma} - \sum_{i=1}^N\sum_{j=1}^p a_{ij}|x|^{1-2\gamma}\Big] > \frac{T}{3}\Big(\int_0^T g(t)\,dt\Big)^2.$$

Then problem equas. (1)—(2) has at least one solution which minimizes φ on H_T^1.

Proof In a similar way to ref. [2], by condition (i), Sobolev's inequality and $\gamma < 1$, one has

$$\Big|\int_0^T [F_1(t,u(t)) - F_1(t,\bar{u})]\,dt\Big| = \Big|\int_0^T\int_0^1 (\nabla F_1(t,\bar{u}+s\tilde{u}(t)),\tilde{u}(t))\,ds\,dt\Big| \leqslant$$
$$\int_0^T\int_0^1 g(t)|\bar{u}+s\tilde{u}(t)|^{\gamma}|\tilde{u}(t)|\,ds\,dt + \int_0^T h(t)|\tilde{u}(t)|\,dt \leqslant 2(|\bar{u}|^{\gamma} + \|\tilde{u}\|_\infty^{\gamma})\|\tilde{u}\|_\infty\int_0^T g(t)\,dt + \|\tilde{u}\|_\infty\int_0^T h(t)\,dt \leqslant$$
$$\frac{3}{T}\|\tilde{u}\|_\infty^2 + \frac{T}{3}|\bar{u}|^{2\gamma}\Big(\int_0^T g(t)\,dt\Big)^2 + 2\|\tilde{u}\|_\infty^{\gamma+1}\int_0^T g(t)\,dt + \|\tilde{u}\|_\infty\int_0^T h(t)\,dt \leqslant$$
$$2\Big(\frac{T}{12}\Big)^{\frac{\gamma+1}{2}}\int_0^T g(t)\,dt\,\|\dot{u}\|_2^{\gamma+1} + \frac{T}{3}|\bar{u}|^{2\gamma}\Big(\int_0^T g(t)\,dt\Big)^2 + \frac{1}{4}\|\dot{u}\|_2^2 + \Big(\frac{T}{12}\Big)^{\frac{1}{2}}\Big(\int_0^T h(t)\,dt\Big)\|\dot{u}\|_2 \tag{3}$$

and it follows from assumption (ii), Wirtinger's inequality and Sobolev's inequality that

$$\Big|\int_0^T [F_2(u(t)) - F_2(\bar{u})]\,dt\Big| = \Big|\int_0^T\int_0^1 (\nabla F_2(\bar{u}+s\tilde{u}(t)),\tilde{u}(t))\,ds\,dt\Big| =$$
$$\Big|\int_0^T\int_0^1 (\nabla F_2(\bar{u}+s\tilde{u}(t)) - \nabla F_2(\bar{u}),\tilde{u}(t))\,ds\,dt\Big| \leqslant \int_0^T\int_0^1 r_1 s|\tilde{u}(t)|^2\,ds\,dt + r_2\int_0^T|\tilde{u}(t)|\,dt \leqslant$$
$$\frac{r_1}{2}\int_0^T|\tilde{u}(t)|^2\,dt + r_2T\|\tilde{u}\|_\infty \leqslant \frac{r_1T^2}{8\pi^2}\|\dot{u}\|_2^2 + \frac{\sqrt{3}r_2T^{\frac{3}{2}}}{6}\|\dot{u}\|_2 \tag{4}$$

for all $u \in H_T^1$.

For $u = (u_1,\cdots,u_N) \in H_T^1$, let

$$\bar{u}_i := \frac{1}{T}\int_0^T u_i(t)\,dt \quad (i = 1,2,\cdots,N),$$

and

$$|\bar{u}| := \Big(\sum_{i=1}^N |\bar{u}_i|^2\Big)^{\frac{1}{2}}.$$

Hence, $\bar{u}_i = u_i(\xi_i)$ for some $\xi_i \in [0,T]$, $i = 1,2,\cdots,N$. Therefore, we have

$$|u_i(t_j)| \leqslant |u_i(t_j) - \bar{u}_i| + |\bar{u}_i| = |u_i(t_j) - u_i(\xi_i)| + |\bar{u}_i| = \Big|\int_{\xi_i}^{t_j}\dot{u}_i(s)\,ds\Big| + |\bar{u}_i| \leqslant T^{\frac{1}{2}}\Big[\int_0^T|\dot{u}_i(s)|^2\,ds\Big]^{\frac{1}{2}} + |\bar{u}_i| \leqslant T^{\frac{1}{2}}\|\dot{u}\|_2 + |\bar{u}| \tag{5}$$

and

$$|u_i(t_j)|^{\gamma_{ij}+1} \leqslant 2^{\gamma_{ij}+1}\Big[T^{\frac{\gamma_{ij}+1}{2}}\|\dot{u}\|_2^{\gamma_{ij}+1} + |\bar{u}|^{\gamma_{ij}+1}\Big] \tag{6}$$

for $i = 1,2,\cdots,N$, $j = 1,2,\cdots,p$.

By assumption (B), equas. (5) and (6) we have

$$\Big|\sum_{i=1}^N\sum_{j=1}^p\int_0^{u(t_j)} I_{ij}(s)\,ds\Big| \leqslant \sum_{i=1}^N\sum_{j=1}^p\int_{\min\{0,u_i(t_j)\}}^{\max\{0,u(t_j)\}} a_{ij}\,ds + \sum_{i=1}^N\sum_{j=1}^p\int_{\min\{0,u_i(t_j)\}}^{\max\{0,u(t_j)\}} b_{ij}\times|s|^{\gamma_{ij}}\,ds \leqslant \sum_{i=1}^N\sum_{j=1}^p a_{ij}\times|u_i(t_j)| + \sum_{i=1}^N\sum_{j=1}^p b_{ij}\times|u_i(t_j)|^{\gamma_{ij}+1} \leqslant$$

$$[T^{\frac{1}{2}}\|\dot{u}\|_2 + |\bar{u}|]\sum_{i=1}^{N}\sum_{j=1}^{p}a_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{p}b_{ij}2^{\gamma_{ij}+1}[T^{\frac{\gamma_{ij}+1}{2}}\|\dot{u}\|_2^{\gamma_{ij}+1} + |\bar{u}|^{\gamma_{ij}+1}] \quad (7)$$

for all $u \in H_T^1$.

It follows from equas. (3), (4) and (7) that

$$\varphi(u) = \frac{1}{2}\|\dot{u}\|_2^2 + \int_0^T[F_1(t,u(t)) - F_1(t,\bar{u})]\mathrm{d}t + \int_0^T F(t,\bar{u})\mathrm{d}t + \int_0^T[F_2(u(t)) - F_2(\bar{u})]\mathrm{d}t + \sum_{i=1}^{N}\sum_{j=1}^{p}\int_0^{\dot{u}(t_j)}I_{ij}(s)\mathrm{d}s \geqslant C_0\|\dot{u}\|_2^2 + C_1\|\dot{u}\|_2^{\gamma+1} - \sum_{i=1}^{N}\sum_{j=1}^{p}b_{ij}2^{\gamma_{ij}+1}T^{\frac{\gamma_{ij}+1}{2}}\|\dot{u}\|_2^{\gamma_{ij}+1} + C_2\|\dot{u}\|_2 + |\bar{u}|^{2\gamma}\left[\frac{1}{|\bar{u}|^{2\gamma}}\int_0^T F(t,\bar{u})\mathrm{d}t - \frac{T}{3}\left(\int_0^T g(t)\mathrm{d}t\right)^2\right] - \sum_{i=1}^{N}\sum_{j=1}^{p}b_{ij}2^{\gamma_{ij}+1}|\bar{u}|^{\gamma_{ij}+1} - \sum_{i=1}^{N}\sum_{j=1}^{p}a_{ij}|\bar{u}|.$$

for all $u \in H_T^1$, where

$$C_0 = \frac{1}{4} - \frac{r_1T^2}{8\pi^2} > 0, C_1 = -2\left(\frac{T}{12}\right)^{\frac{\gamma+1}{2}}\int_0^T g(t)\mathrm{d}t,$$

$$C_2 = -\left(\frac{T}{12}\right)^{\frac{1}{2}}\left(r_2T + \int_0^T h(t)\mathrm{d}t\right) - \sum_{i=1}^{N}\sum_{j=1}^{p}a_{ij}T^{\frac{1}{2}}.$$

Set

$$\varphi_1(s) = C_0s^2 + C_1s^{\gamma+1} + C_2s - \sum_{i=1}^{N}\sum_{j=1}^{p}b_{ij}2^{\gamma_{ij}+1}T^{\frac{\gamma_{ij}+1}{2}}s^{\gamma_{ij}+1}, s \geqslant 0;$$

$$\varphi_2(\tau) = \int_0^T F(t,\tau)\mathrm{d}t - \frac{T}{3}\left(\int_0^T g(t)\mathrm{d}t\right)^2|\tau|^{2\gamma} - \sum_{i=1}^{N}\sum_{j=1}^{p}b_{ij}2^{\gamma_{ij}+1}|\tau|^{\gamma_{ij}+1} - \sum_{i=1}^{N}\sum_{j=1}^{p}a_{ij}|\tau|, \quad \tau \in \mathbb{R}.$$

Since $r_1 < \frac{2\pi^2}{T^2}$, $\gamma + 1 < 2$, and $\gamma_{ij} + 1 < 2$, by (iii), there exists $K_0 > 0$ such that

$$\varphi_1(s) \geqslant \varphi(0) - \inf_{\tau\in\mathbb{R}}\varphi_2(\tau), \text{for } s \geqslant K_0,$$

and

$$\varphi_2(\tau) \geqslant \varphi(0) - \inf_{s\geqslant 0}\varphi_1(s), \text{for } |\tau| \geqslant K_0.$$

By Wirtinger's inequality, one has

$$\|u\|^2 = \int_0^T|\tilde{u}(t) + \bar{u}|^2\mathrm{d}t + \int_0^T|\dot{u}(t)|^2\mathrm{d}t \leqslant 2\int_0^T|\tilde{u}(t)|^2\mathrm{d}t + 2T|\bar{u}|^2 + \int_0^T|\dot{u}(t)|^2\mathrm{d}t \leqslant \left[\frac{T^2}{2\pi^2} + 1\right]\|\dot{u}\|_2^2 + 2T|\bar{u}|^2.$$

Take $K = \left[1 + \frac{T^2}{2\pi^2} + 2T\right]^{1/2}K_0$. If $\|u\| \geqslant K$, then we have

$$\left[\frac{T^2}{2\pi^2} + 1\right]\|\dot{u}\|_2^2 + 2T|\bar{u}|^2 \geqslant K^2,$$

which implies that

$$\|\dot{u}\|_2 \geqslant K_0, \text{or } |\bar{u}| \geqslant K_0.$$

There are two cases to consider.

Case 1. If $\|\dot{u}\|_2 \geqslant K_0$, then we have

$$\varphi(u) \geqslant \varphi_1(\|\dot{u}\|_2) + \varphi_2(\bar{u}) \geqslant \varphi(0) - \inf_{\tau\in\mathbb{R}}\varphi_2(\tau) + \varphi_2(\bar{u}) \geqslant \varphi(0).$$

Case 2. If $|\bar{u}| \geqslant K_0$, then we have

$$\varphi(u) \geqslant \varphi_1(\|\dot{u}\|_2) + \varphi_2(\bar{u}) \geqslant \varphi_1(\|\dot{u}\|_2) + \varphi(0) - \inf_{s\geqslant 0}\varphi_1(s) \geqslant \varphi(0).$$

Hence we have

$$\varphi(u) \geqslant \varphi(0),$$

for all $\|u\| \geqslant K$.

That is,

$$\inf_{\|u\|\geqslant K}\varphi(u) \geqslant \varphi(0),$$

which means

$$\inf_{u\in H_T^1}\varphi(u) = \inf_{\|u\|<K}\varphi(u).$$

So φ has a bounded minimizing sequence. By Theorem 1.1 in ref. [3], the proof is completed.

Remark 2 Theorem 1 without impulses improves Theorem 2.2 in ref. [4].

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An Hybrid Genetic Algorithm for Solving $L(2,1)$-Label of Graph

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[**Abstract**] A hybrid genetic algorithm for solving the $L(2,1)$-label problem based on combining the Floyd algorithm, greedy algorithm and tenetic algorithm is presented. Also the simulation results show that compared with genetic algorithm, the hybrid algorithm has converges faster and can solve the $L(2,1)$-label of a given graph fleetly.

[**Key words**] $L(2,1)$-label greedy altorithm hybrid algorithm

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Periodic Solutions for Second Order Hamiltonian Systems with Impulses

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[**Abstract**] By using the least action principle, a new existence theorem is obtained for periodic solutions of nonautonomous second order Hamiltonian systems with impulses in the derivative without the coerciveness assumption. Some related results are improved.

[**Key words**] periodic solution impulses the least action principle

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A New Class of Biased Estimation in the Restricted Linear Regression Model

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[**Abstract**] To restricted liner regression model, a new biased estimation $\beta_R^*(k) = (kM + I)^{-1}\beta_R^*$ and $\beta_R^*(k)$ superior to β_R^* in terms of mean squares error matrix are showed.

[**Key words**] biased estimation generalized least squares estimation mean squares error matrix