第10卷 第14期 2010年5月
1671-1815(2010)14-3344-05
科 学 技 术 与 工 程
Science Technology and Engineering
Vol. 10 No. 14 May 2010
© 2010 Sci. Tech. Engng.

贝叶斯框架下的模糊图像盲去卷积算法

肖 宿 韩国强 沃 焱 姚浩伟[1]
(华南理工大学 计算机科学与工程学院,广州 510006;深圳市华仁达电子有限公司[1],深圳 518040)

摘 要 为了去除图像模糊的同时,保持图像边缘等细节信息,需要对原始图像和点扩散函数进行准确的估计。在贝叶斯框架下,基于总变分模型,建立原始图像和点扩散函数的先验模型,同步估计原始图像和点扩散函数。对于总变分模型不可微分的问题,在不影响速度的前提下,用迭代重加权范数算法处理该问题。基于共轭分布理论,提出以伽马分布作为未知参数的先验模型,准确估计参数。实验表明该算法在对原始图像、点扩散函数和参数准确估计的基础上,成功地解决了模糊图像的盲去卷积问题,算法的速度和效果都得到了改进。与同类算法相比,本文提出的算法具有一定优势。

关键词 图像盲去卷积 贝叶斯框架 先验模型 总变分模型 数值计算
中图法分类号 TP751.1; **文献标志码** A

It has a long history to research the blind source separation which in image processing, can be utilized to solve the problem of blind image deconvolution. The term blind refers to the Point Spread Function (PSF) of the imaging system is totally-unknown. Currently, there mainly exist two kinds of blind image deconvolution algorithms: the stochastic [1,2] and the deterministic [3]. The latter can be treated as the constrained optimization, which the constraints can be brightness preserving assumption or smoothing assumption of the PSF; The stochastic algorithms treat the unknown variables as the random variables, and deconvolution can implemented using the statistical properties of the original image and the prior knowledge of the imaging system. Compared with the deterministic algorithms, the critical point is to model the original image, the PSF and the unknown parameters using the prior models.

Therefore, we firstly establish the prior models of the original image, the PSF and the unknown parameters. Based on the Bayesian inference, the optimal original image, the PSF and the parameters are obtained. Because the prior model of the original image is the total variation model which is non-differentiable, the Iteratively Reweighted Norm method is introduced to solve this problem.

2010年3月15日收到 国家自然科学基金(60573019)、国家科技支撑计划(X2JSB1080010)资助

第一作者简介:肖 宿(1982—),男,安徽淮北人,博士研究生,研究方向:图像恢复技术。E-mail:SmartWindows@qq.com。

1 Prior models

The degradation model which mirrors the process that the original image degraded into the observed image can be expressed as follow:

$$g = Hu \tag{1}$$

where $g \in R^{N\times 1}$ represents the lexicographically ordered observed image; $u \in R^{N\times 1}$ represents the lexicographically ordered original image; $H \in R^{N\times N}$ is the degradation matrix which is formed by the PSF.

The blind image deconvolution is the inverse problem, and in the Bayesian framework, it is equal to the optimization problem as follow:

$$(\hat{u},\hat{h},\hat{\alpha},\hat{\beta}) = \underset{u,h,\alpha,\beta}{\arg\max}\, p(\hat{u},\hat{h},\hat{\alpha},\hat{\beta} \mid g) \tag{2}$$

From the conditional probability formula below:

$$p(u,h,\alpha,\beta \mid g) = \frac{p(u,h,\alpha,\beta,g)}{p(g)} \propto p(u,h,\alpha,\beta,g) \tag{3}$$

where the $p(g)$ is constant with knowing the observed image, we can obtain:

$$(\hat{u},\hat{h},\hat{\alpha},\hat{\beta}) = \underset{u,h,\alpha,\beta}{\operatorname{argmax}}\, p(u,h,\alpha,\beta,g) \tag{4}$$

where the joint probability can be decomposed as $p(u, h, \alpha, \beta, g) = p(\alpha)p(\beta)p(u|\alpha)p(h/\beta)p(g|u,h)$. $p(\alpha)$, $p(\beta)$, $p(u \mid \alpha)p(h/\beta)$ and $p(g \mid u,h)$ are called prior models.

1.1 Observation model

A commonly used observation model can be expressed as:

$$p(g \mid u,h) \propto \exp[-F(u,h)] = \exp(-\|g - Hu\|_2^2) \tag{5}$$

where $F(u,h)$ represents data fidelity which measures the closeness between Hu and g in the least squares sense. The definition of the observation model shown as formula (5) is based on the assumption that the noise in the observed image is Gaussian, while the statistics shows that the noise is more likely to be Laplacian [4]. So we define the observation model as follow:

$$p(g \mid u,h) \propto \exp[-F(u,h)] = \exp[-\|g - Hu\|_1] \tag{6}$$

1.2 Prior model of the original image

In Bayesian framework, image deconvolution depends on the efficient image modeling which can be carried out though Markov random field, graph model, information theory. *et al*, and the prior model of the original image can be thought as the constraint on image deconvolution. Among the image models, the total variation model is most widely used for its excellent edge-preserving ability [5]. We utilize the total variation model to define the following prior model of the original image:

$$p(u \mid \alpha) \propto \alpha^{N/2}\exp[-\alpha TV(u)] = \alpha^{N/2}\exp[-\alpha \| \sqrt{(\Delta_x u)^2 + (\Delta_y u)^2} \|_1] \tag{7}$$

where $\Delta_x \in R^{N \times N}$, $\Delta_y \in R^{N \times N}$ denote the first-order horizontal difference and vertical difference separately; $\alpha > 0$ is called the unknown model parameter.

1.3 Prior model of PSF

The PSF can also be treated as an image which has the edges. Therefore, the prior model of the PSF can be defined as:

$$p(h \mid \beta) \propto \beta^{N/2}\exp[\beta TV(h)] = \beta^{N/2}\exp(-\beta \| \sqrt{(\Delta_x h)^2 + (\Delta_y h)^2} \|_1) \tag{8}$$

where the support of PSF is expanded by padding zeros outside of its original support; β is the model parameter.

1.4 Parameter models

According to the concept of the conjugate distributions [6], the prior distributions and the posterior distributions of parameters have the same form, and we can find that Gamma distribution belongs to the conjugate distributions. So it is used as the prior models of the unknown model parameters:

$$p(\omega) \propto \omega^{m_\omega - 1}\exp(-n_\omega \omega) \tag{9}$$

where $m_\omega > 0$, $n_\omega > 0$ are const and denote the shape hyperparameter and the scale hyperparameter separately; $\omega = \{\alpha, \beta\}$. The initial value of the model parameter is $\omega^{(1)} = m_\omega / n_\omega$.

2 Solution for L_1-optimization

In the Bayesian framework, the inference based on maximizing the posterior is equal to the following L_1-optimization:

$$\min\{F(u) + \alpha TV(u) + \beta TV(h)\} = \min\{\|g - Hu\|_1 + \alpha \| \sqrt{(\Delta_x u)^2 + (\Delta_y u)^2} \|_1 + \beta \| \sqrt{(\Delta_x h)^2 + (\Delta_y h)^2} \|_1\} \tag{10}$$

It is obviously that due to the existence of L_1 norm which is non-differentiable, a numerical algorithm certain is needed for the optimization above. Here we adopt the Iteratively Reweighted Norm (IRN) [7] to implement the numerical calculation of the L_1-optimization. IRN minimizes the L_1 norm by approximating it, within an iterative scheme, by a weighted norm. So $F(u,h)$, $TV(u)$ and $TV(h)$ can be expressed using IRN:

$$\begin{cases} Q_F^{(k)}(u,h) = \frac{\beta}{2}\|(W_F^{(k)})^{1/2}(g - Hu)\|_2^2 + \frac{1}{2}F(u^{(k)},h^{(k)}) \\ Q_u^{(k)}(u) = \frac{\alpha}{2}\|(W_u^{(k)})^{1/2}\Delta u\|_2^2 + \frac{\alpha}{2}TV(u^{(k)}) \\ Q_h^{(k)}(h) = \frac{\beta}{2}\|(W_h^{(k)})^{1/2}\Delta h\|_2^2 + \frac{\beta}{2}TV(h^{(k)}) \end{cases} \tag{11}$$

where Δ, $W_F^{(k)}$, $W_u^{(k)}$ and $W_h^{(k)}$ can be defined:

$$\Delta = \begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix},\ W_F^{(k)} = \operatorname{diag}(\tau_F(g - H_u^{(k)}));$$

$$W_u^{(k)} = \begin{pmatrix} \Omega_u^{(k)} & 0 \\ 0 & \Omega_u^{(k)} \end{pmatrix},\ W_h^{(k)} = \begin{pmatrix} \Omega_h^{(k)} & 0 \\ 0 & \Omega_h^{(k)} \end{pmatrix}.$$

where $\Omega_u^{(k)}$ and $\Omega_h^{(k)}$ have the following definitions:

$$\Omega_u^{(k)} = \operatorname{diag}(\tau_u((\Delta_x u)^2 + (\Delta_y u)^2)),$$
$$\Omega_h^{(k)} = \operatorname{diag}(\tau_h((\Delta_x h)^2 + (\Delta_y h)^2)).$$

The threshold functions τ_u, τ_h and τ_F have the same type as follow:

$$\tau(z) = \begin{cases} |z|^{-1}, & \text{if } |z| > \varepsilon \\ \varepsilon^{-1}, & \text{if } |z| \leqslant \varepsilon \end{cases} \tag{12}$$

3 Description of proposed algorithm

According to formula (2), the estimation of each unknown variable can be written uniformly as follow:

$$\hat{\xi} = \underset{\xi}{\operatorname{argmax}}\, p(\xi, \hat{\theta}_\xi, g) \tag{13}$$

where $\xi \in \{u, h, \alpha, \beta\}$; $\theta = \{u, h, \alpha, \beta\}$; θ_ξ denotes the set of reft elements with ξ excluded from θ. Therefore, the proposed algorithm can be described as follow:

Initial values: $\alpha^{(1)}$, $\beta^{(1)}$, $u^{(1)}$ and $h^{(1)}$

For $k = 1, 2, \cdots$, until stopping riteration

step 1: calculation: $W_F^{(k)}$, $W_u^{(k)}$ and $W_h^{(k)}$

step 2: $u^{(k+1)} = \arg\max_u[\ln p(u|\alpha^{(k)}) + \ln p(g|u, h^{(k)})] \Rightarrow u^{(k+1)} = [\alpha^{(k)}\Delta^{\mathrm{T}}W_u^{(k)}\Delta + (H^{(k)})^{\mathrm{T}}W_F^{(k)}H^{(k)}]^{-1} \times (H^{(k)})^{\mathrm{T}}W_F^{(k)}g$ (14)

step 3: $h^{(k+1)} = \arg\max_h[\ln p(h|\beta^{(k)}) + \ln p(g|u^{(k+1)}, h)] \Rightarrow$

$$h^{(k+)} = [\beta^{(k)}\Delta^{\mathrm{T}}W_h^{(k)}\Delta + (U^{(k+1)})^{\mathrm{T}}W_F^{(k)}U^{(k+1)}]^{-1}(U^{(k+1)})^{\mathrm{T}}W_F^{(k)}g \tag{15}$$

step 4: $\alpha^{(k+1)} = \arg\max_\alpha[\ln p(\alpha) + \ln p(u^{(k+1)} | \alpha)] \Rightarrow$

$$(\alpha^{(k+1)})^{-1} = \kappa_\alpha \frac{1}{\alpha^{(1)}} + (1 - \kappa_\alpha)\frac{2TV[u^{(k+1)}]}{N-2} \tag{16}$$

$\beta^{(k+1)} = \arg\max_\beta[\ln p(\beta) + \ln p(h^{(k+1)} | \beta)] \Rightarrow$

$$(\beta^{(k+1)})^{-1} = \kappa_\beta \frac{1}{\beta^{(1)}} + (1 - \kappa_\beta)\frac{2TV[h^{(k+1)}]}{N-2} \tag{17}$$

step 5: if $\dfrac{\|u^{(k+1)} - u^{(k)}\|_2^2}{\|u^{(k)}\|_2^2} \leqslant 10^{-5}$

then output $u^{(k+1)}$

else $k = k+1$

return to step 1

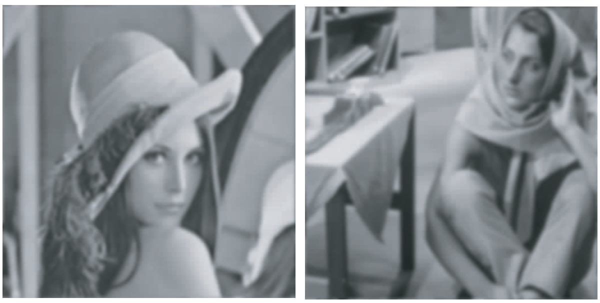
---

where $\kappa_\alpha = m_\alpha/(m_\alpha + N/2 - 1)$, $\kappa_\beta = m_\beta/(m_\beta + N/2 - 1)$ denote confidence parameters and take values in the interval $[0, 1)$.

4 Experimental results

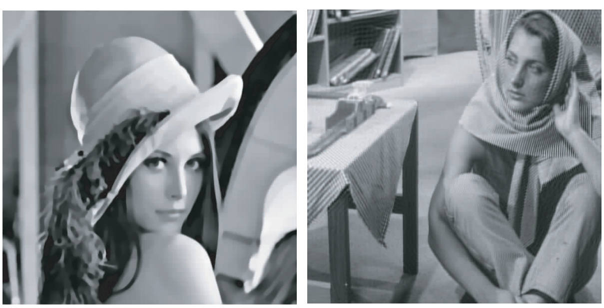
Several experiments have been carried out for the proposed algorithm using several degraded images. The proposed algorithm is compared with MM method [8] in which the PSF is known. Since MM method assumes exact knowledge of the PSF, it provides an upper bound of the achievable quality by the blind deconvolution methods. We use the improvement in signal－to－noise ratio defined as $\mathrm{ISNR} = 10\lg_{10}(\|u - g\|_2^2 / \|u - u_{\mathrm{est}}\|_2^2)$ to measure the quality of the restored images, where u_{est} represents the estimated original images.

Experiment I: The original image Lena and Barbara are used and they were blurred by a Gaussian shaped PSF to obtain the degraded images as shown in fig. 1. The restored results obtained by using the proposed algorithm and MM method are shown in fig. 2. Table 1 shows the iteration number and time needed for the proposed algorithm and MM method to restore the original images.



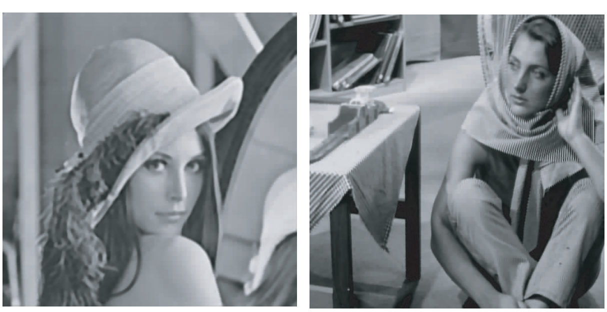
(a) Lena, BSNR = 30 dB　　(b) Barbara, BSNR =30 dB

Fig. 1　Blurred images



ISNR = 4.0 dB　　ISNR =3.8 dB

(a) Results obtained by proposed algorithm



ISNR = 3.3 dB　　ISNR = 2.9 dB

(b) Results obtained by MM method

Fig. 2　Restored results

Table 1　Iterations and time costs

Blurred Images	Algorithms	Iteration	Time/s
Lena	Proposed	13	25.31
	MM	41	123.45
Barbara	Proposed	21	34.89
	MM	52	143.21

Experiment II: The original image Lena and Barbara are used and they were blurred by a uniform shaped PSF to obtain the degraded images. The restored results obtained by using the proposed algorithm and MM method are shown in table 2.

Table 2　Uniform-blurred images and their deconvolution results

Blurred Images	Algorithms	Iteration	Time /s	$\frac{\text{ISNR}}{\text{dB}}$
Lena	Proposed	7	14.22	6.4
	MM	31	87.14	5.3
Barbara	Proposed	25	40.32	4.6
	MM	63	166.98	3.6

The experiments show that the blurs can be removed successfully by using the proposed algorithm, with satisfactory speed. The proposed algorithm shows better performance than MM method which uses the conjugate gradient method to estimate the original image.

5　Conclusions

Based on the Bayesian framework, a novel algorithm has been proposed for simultaneous estimation of the original image,The PSF and the unknown parameters in blind deconvolution problems. Using this algorithm, we can approximate the posterior distributions of the original image and the PSF, as well as the unknown parameters. The proposed algorithm has been analyzed, validated and compared with synthetic data.

References

1 Tonazzini A, Bedini L, Salemo E. A Markov model for blind image separation by a mean-field EM algorithm. IEEE Transactions on Image Processing, 2006;15(2): 473—482

2 Ichir M M, Mohammad-Djafari A. Hidden Markov models for wavelet-based blind source separation. IEEE Transactions on Image Processing, 2006;15(7): 1887—1899

3 Sroubek F, Flusser J. Multichannel blind iterative image restoration. IEEE Transactions on Image Processing, 2003;12(9): 1094—1106

4 Farsiu S, Elad M, Milanfar P. Multi-frame demosaicing and super-resolution of color images. IEEE Transactions on Image Processing, 2006;15(1): 141—159

5 Chan T F, Shen J. Image processing and analysis: variational, PDE, wavelet, and stochastic methods. Philadelphia, PA: SIAM, 2005

6 Berger J O. Statistical decision theory and bayesian analysis. 2nd Edition. New York: Springer-Verlag, 1985

7 Rodriguez P, Wohlberg B. Efficient minimization method for a generalized total variation functional. IEEE Transactions on Image Processing, 2009;18(2): 322—332

8 Oliveira J P, Bioucas-dias J M, Figueiredo M A T. Adaptive total variation image deblurring: a majorization-minimization approach. Elsevier Journal of Signal Processing, 2009;89(9): 1683—1693

Bayesian Framework Based Blind Deconvolution Algorithm for Blurred Images

XIAO Su, HAN Guo-qiang, WO Yan, YAO Hao-wei[1]

(School of Computer Science and Engineering, South China University of Technology, Guangzhou 510006, P. R. China; Shenzhen Fanwo Electronic Co., Ltd[1], Shenzhen 518040, P. R. China)

[**Abstract**] For the purpose of deblurring the blurred images without the loss of the detailed information as edges, it was necessary to estimate the original image and point spread functions accurately. A Bayesian framework is proposed based algorithm which used the total variation model to describe the original image. The total variation could preserve the edges of deblurred images, while it was non-differentiable. Therefore, the Iteratively Reweighted Norm method is utilized to solve this problem. Based on the conjcept of the prior distribution, the Gamma distribution was introduced as the prior models of the unknown model parameters. The experimental results show the competitive performance of the proposed algorithm. With the accurate estimation of the original image and unknown model parameters, the blind deconvolution can be implemented successfully. The speed of the proposed algorithm and the results of the deconvolution are all improved obviously. Compared with the similar algorithm, the proposed algorithm has some advantages.

[**Key words**] image blind deconvolution Bayesian framework prior models total variation model numerical calculation