## 基于 DE—APVIOBPCS 模型的零 库存偏移变体分析

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摘 要 首先利用控制工程领域的方法描述了单阶供应链中基于 AR(1)需求和最小均方误差预测的 DE-APVIOBPCS 模型。通过应用终值定理可知,不论提前期的估计准确与否,系统的终值偏差(即库存偏移)都将始终存在。为了消除此系统的固有偏差并保持合理的系统动态性,作者提出了一种基于 DE-APVIOBPCS 模型的零库存偏移变体。由方差放大分析可知,在新的变体模型中采用保守的提前期估计值总是比较有利。分析了零库存偏移变体模型的稳定性条件,并在提前期错误估计的假设前提下通过仿真分析举例说明了新模型的一些优点。

**关键词** 提前期估计 库存偏移 方差放大 补给策略 z变换中图法分类号 F253.4: 文献标志码 A

The use of control engineering approaches to solve production and inventory problems has been well studied. Coyle<sup>[1]</sup> identified the Inventory and Order Based Production Control System (IOBPCS) model, which laid the foundation of a generic family of production control systems. To formalize the decision making process by utilizing simple, yet robust, algorithms, Sterman [2] suggested that a decision making model which will allow suitable consideration of the pipeline and lead to stable dynamic behavior. This approach is known as Automatic Pipeline, Variable Inventory and Order Based Production Control System (APVIOBPCS), which was examined by John [3] via mathematical and simulation analysis. Disney and Towill reviewed the IOBPCS family of decision support systems in Reference [4]. The equivalence of APVIOBPCS model and the general Order-Up-To (OUT) policy was subsequently established

2009 年 9 月 7 日收到 国家自然科学基金(50878089) 资助 第一作者简介:何佳宁(1982—),湖南临湘,博士研究生,研究方向: 供应链生产与库存系统。E-mail;he jia\_1982@163.com。 by Dejonckheere *et al.* <sup>[5]</sup>. The latter was proposed to decrease the bullwhip effect and generate smoothing orders.

This type of ordering policies is under the assumption of having an accurate estimate of the production and delivery lead-times. It's readily shown they'll suffer from inventory drift if the lead-time estimate is not always equal to the actual one [3]. Inventory drift is used to describe the phenomena that, over time, inventory levels do not "lock on" to target levels when a step change in the consumption rate has occurred [4]. This will definitely affect the net stock level and thus system dynamics if we can't find a way to fix it. It seems that the most effective solution is the continuous monitoring of actual lead-times. However this requires a significant amount of management effort without theoretical support on the stability.

After searching the related literatures, much fewer articles about inventory drift were found than what we expected. Two seminal papers, written by John<sup>[3]</sup> and Disney *et al.* <sup>[4]</sup> respectively, both worked

on the APVIOBPCS with independent and identically distributed (i. i. d.) demand and exponential smoothing forecasting. John [3] first examined the existence of inventory drift using the Final Value Theorem (FVT) when the lead – time estimation is not accurate. Then Disney et al. [4] presented a novel Estimated Pipeline Variable Inventory and Order Based Production Control System (EPVIOBPS) to eliminate the inventory deficit instead of continuous monitoring of actual lead-times. When concerning to different demand patterns and forecasting methods, sometimes the inventory deficit is inherent even though the accurate lead-time is provided.

#### 1 DE-APVIOBPCS model

The single-echelon supply chains consisting of a retailer and a manufacturer is eonsidered. The first order AR (1) (auto-regressive) demand pattern is assumed and the demand  $D_t$  faced by the retailer in period t is shown in equation (1).

$$D_t = \mu + \rho(D_{t-1} - \mu) + \varepsilon_t \tag{1}$$

where  $\mu$  is the mean of demand,  $\varepsilon_t$  is an i.i.d. normally distributed random error, and  $\rho$  is the first – order autocorrelation coefficient which is subjected to  $|\rho| < 1$ . The Minimum Mean Square Error (MMSE) forecasting scheme is optimal when demand is AR(1) process as it explicitly takes the correlative demand structure into account and can minimize the inventory cost concerned [6].

A special case of APVIOBPCS as DE-APVIOB-PCS is named, which emphasizes the system stability by setting the smoothing parameters or the controllers of net stock and work-in-progress (WIP) loop to be equal <sup>[7]</sup>. Because the equivalence of APVIOBPCS model and general OUT policy<sup>[5]</sup>, the corresponding OUT form for DE-APVIOBPCS can be obtained, which is given in equation (2) and is well-known for its

highly desirable ability to keep system robust and reduce the order variability [8]

$$O_{t} = D_{t+L} + \frac{1}{T_{i}} (TNS_{t} - NS_{t}) + \frac{1}{T_{i}} (DWIP_{t} - WIP_{t})$$
(2)

where  $D_{t+L}$  is the MMSE forecast for the demand in period t+L.  $NS_t$  and  $WIP_t$  are not stock and work in progress at period t+L. respectively.  $TNS_t = a\hat{D}_{t+L}$  is a target net stock level, which is updated every period according to the new demand forecast.  $DWIP_t$  is the desired WIP level, which is updated every period as well,  $DWIP_t = \sum_{i=1}^{T_p} \hat{D}_{t+i}$ .  $T_i$  is the smoothing parameter or controller of the decision rule.

The policy in equation (2) can be described in words as "ordering quantities are set equal to the sum of forecasted demand, an equal fraction of the discrepancy of finished goods net stock and on – order position discrepancy". The corresponding block diagram is given in the fig. 1. Please note that an estimate  $T'_p$  of the actual lead time  $T_p$  in the estimation of order-up-to level is used, but the lead-time estimate  $T'_p$  doesn't affect the actual lag in production and delivery. So the actual delay of  $T_p$  still exists in the recursion of net stock and WIP.  $T_p = 3$  is set throughout this paper to quantify and compare the variance amplification. Other values can be chosen and the same conclusion will be found after the following analysis.

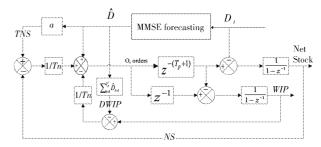


Fig. 1 Block diagram for DE-APVIOBPCS model in single-echelon supply chains

#### 2 The analysis of inventory drift

In order to determine the behavioral boundary conditions for a stable system, the Initial and Final Value Theorems (IVT and FVT, respectively) are employed. This approach was presented by John<sup>[3]</sup> first in the form of continuous Laplace transform. Here them in z-domain are restated.

IVT: 
$$\lim_{t \to 0} \{ f(t)i(t) \} = \lim_{z \to 0} \{ (1 - z^{-1})F(z)I(z) \}$$
(3)

FVT: 
$$\lim_{t\to\infty} \{f(t)i(t)\} = \lim_{z\to 1} \{(1-z^{-1})F(z)I(z)\}$$
(4)

where the unit step input  $I(z) = 1/(1-z^{-1})$  is used and F(z) represents the corresponding transfer function. To apply IVT and FVT to DE-APVIOB-PCS, i. i. d. random error  $\varepsilon_t$  in equation (1) as the input of the whole system is treated normally. The net stock transfer function of DE-APVIOBPCS over  $\varepsilon_t$  is shown in equation (5).

$$\begin{split} \frac{N_{t}S}{\varepsilon_{t}} &= \left[ -\rho^{1+T_{p}}(-1+a+T_{i}) + \rho^{2+T_{p}}(a+T_{i}) + \right. \\ &\left. z(1+z+z^{2}+T_{i}z^{3}) - \rho(1+z+z^{2}+z^{3}+T_{i}z^{4}) \right] \left[ (-1+\rho)(1+T_{i}(-1+z))(\rho-z)z^{2} \right]^{-1} \end{split} \tag{5}$$

The above theorems have been applied to equation (5) and the results are shown in Table 1. The Initial Value of infinity indicates that the responses of the variable input can match those of the previous unsteady state at period t=0. In other words, this DE-APVIOBPCS model has the ability to deal with the unstable initial net stock level. With determinate parameters value, the Final Value is always a constant, which shows that there might be a final value offset  $(i.\ e.\ inventory\ drift)$  even though the estimate  $T'_p$  is equal to actual lead – time 3.

Table 1 Use of IVT and FVT for Net Stocks when  $T_p = 3$ 

| Initial | Final Value   |  |
|---------|---|--|
| Value   |   |  |
| ∞       | $\frac{-3 - T_i + \rho(4 + T_i) + \rho^{1 + T_p}(-1 + a + T_i) - \rho^{2 + T_p}(a + T_i)}{(-1 + \rho)^2}$ |  |

**Insight** Keeping the inventory level stable is important because the inventory offset can make net stock variance dramatically large when the lead – time is variable and the frequent estimation error is inevitable. It is essential to develop a replenishment policy which can not only keep the system variances acceptable but also can eliminate the inherent inventory drift. Apparently the DE-APVIOBPCS model with AR(1) demand and MMSE forecasting method can not address this issue.

#### 3 Zero inventory drift variant

According to table 1, the Final Value shows that an offset can be eliminated with appropriate parameter setting.  $T_i$  is set as in equation (6) to gain the benefit of no inventory drift in new policy, which will be called zero inventory drift variant thereafter.

$$FV_{NS} = 0 \Rightarrow T_{i} = \frac{-3 + 4\rho - \rho^{1+T'_{p}} + a\rho^{1+T'_{p}} - a\rho^{2+T'_{p}}}{(-1 + \rho)(-1 + \rho^{1+T'_{p}})}$$
(6)

The parameter *a* is not solely used to determine the desired net stock level. It is vreated as a key parameter to ensure no inventory offset even though the lead – time is varying over time and misidentification occurred frequently.

The variance amplification of orders (i.e. bull-whip effect) and net stocks are quantified by using the frequency response technique stated in Reference [5]. Simple inspection of fig. 2 and fig. 3 reveal following points.

(1) Zero inventory drift variant will bring up some instability issues when the value of a is small. This will be discussed later in stability analysis.

- (2) Both the order and the net stock variance amplification ratio will lock on a constant value when the value of a is big enough. But the smallest estimation of the lead-time is the winner (i. e. the smallest value of ratio) when the value of a is relatively small but still keep the system stable.
- (3) Misidentification of the lead time has no effect on the variance amplification when the value of a is big enough. In other words, both the order and the net stock variance amplification ratios will converge to a constant value respectively whatever the lead-time estimation is selected.

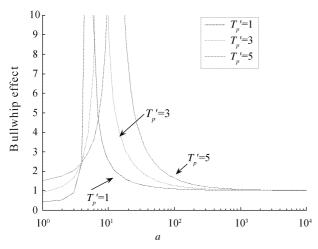


Fig. 2 Bullwhip effect based on zero inventory drift variant policy when  $T_p=3$  and  $\rho=0.7$ 

Thus estimating the lead-time conservatively works well in new policy, since the order and net stock variance amplification ratios can both be decreased when the value of a is relatively small and stay the same when the value of a is big enough. Besides no inventory drift, another very valuable attribute in new policy is both variance amplification ratios will decrease as the value of a increases. This attribute distinguish new policy from other replenishment policies existing. Usually bullwhip effect and net stock variance amplification ratio are against to each other. Such feature draws a lot of attention in research to make a trade-off between them.

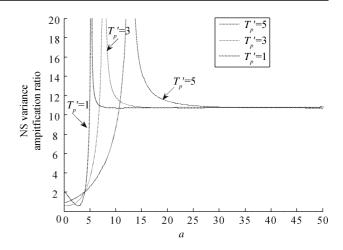


Fig. 3 Net stock variance amplification ratio based on zero inventory drift policy when  $T_p = 3$  and  $\rho = 0.7$ 

# 4 Stability criteria for zero inventorydrift policy

A challenge associated withzero inventory drift variant is the issue of stability (see fig. 2 and fig. 3). We should be more cautions in setting the smoothing parameters concerned with feedback loop. It is particularly important to understand system instability, as in such cases the system response to any change in input will result in uncontrollable oscillations of increasing amplitude and apparent chaos ensuing in the supply chain. In this section the method combined with Tustin Transformation and Routh – Hurwitz stability criterion will be used to determine the limiting condition for system stability. For more detailed explanation of stability analyses please refer to reference [9].

Three examples of parameter plane demonstration of stable and unstable regions are shown in fig. 4. Please note that the logarithm ordinate in the graph is used. It's clear that sometimes it might not be able to set the value of a big enough to satisfy the stability requirement when the demand pattern coefficient is very small. Fortunately that less correlated demand pattern is uncommon in practice at

- all. Simple inspection of fig. 4 reveals that:
- (1) Larger stable zone will be obtained when estimating the lead-timeconservatively, and vice versa.
- (2) The smaller the demand auto corretation coefficient  $\rho$  is the bigger the unstable range of value a will be, and vice versa.

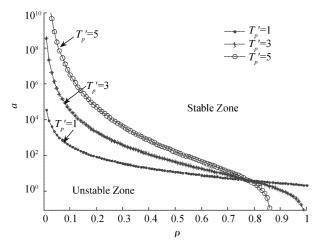


Fig. 4 Parameter plane demonstration of stability based on zero inventory drift policy when  $T_p = 3$ 

#### 5 Simulation experiment

Another valuable attribute on zero inventory driftvariant is that it can works very well when the estimation of the lead-time is treated as a random variable. But this is not the case for DE-APVIOB-PCS model. This advantage through spreadsheet simulation can be proved easity. 900 periods simulation is made for both DE-APVIOBPCS model and zero inventory drift variant. The estimation of the lead-time is assumed to be 2 in the first 300 periods, 4 in the second 300 periods and the actual lead-time 3 in last 300 periods. Any other combination of the lead-times is apparently feasible and the same conclusion will be obtained.

The same demand pattern with  $\rho=0.7$ ,  $\mu=100$  and  $\sigma_e^2=10$  is assumed in both policies. The value of a in zero inventory drift policy is chosen to be 1 000 casually.  $NS_0=a\mu$  is set in zero inventory drift variant to obtain zero target net stock level in our simulation.

Other value of  $NS_0$  is also feasible to meet a different net stock level.  $T_i = 4$  is set in DE-APVIOBPCS model to keep the bullwhip effect around 1. And a is 0 to obtain the same target net stock level as in zero inventory drift variant. Some sample simulation plots (100 periods) of the orders and net stocks for both policies are depicted in fig. 5 and fig. 6.

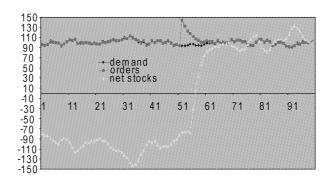


Fig. 5 Generated sample demand, orders and net stocks for DE-APVIOBPCS model

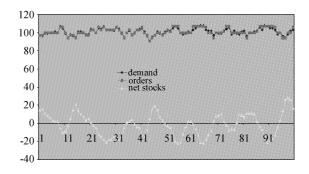


Fig. 6 Generated sample demand, orders and net stocks for zero inventory drift policy

Table 2 Simulation of DE-APVIOBPCS and it's zero inventory drift variant

| Measure     | DE-APVIOBPCS | Zero inventory |  |  |
|-------------|--------------|----------------|--|--|
|             |              | drift policy   |  |  |
| Bullwhip    | 1.22         | 1.14           |  |  |
| NS variance | 333.16       | 22.34          |  |  |
| Fill-rate   | 65.80%       | 96.74%         |  |  |

The average result of running spreadsheet simulation in 100 times is reported in table 2. We have elected to use the "fill-rate" as a suitable customer service level (CSL) metric<sup>[10, 11]</sup>. From table 2, we can clear-

ly prove the strength of zero inventory drift variant overcoming the DE-APVIOBPCS model when lead time is variable and the estimation error is inevitable.

#### 6 Conclusion

The DE-APVIOBPCS model with MMSE forecast for AR(1) demand in a single-echelon supply chain has first been described using block diagram and z-transform tool. After applying the Final Value Theorem, that is demonstrated there must be a final value offset (i. e. inventory drift) even though the actual lead-time is known. Thus the zero inventory drift variant to eliminate the inherent inventory drift is developed and the system variances acceptable are kept. The analysis of the variance amplification suggests the lead-times should be estimated conservatively in supply chains based on the zero inventory drift variant. The general stability condition for new policy has been evaluated by example for three cases.

Two most valuable attributes about new zero inventory drift policy are stated. One is both order and net stock variances decrease as the value of a increases, which distinguish new policy from other policies. The other is that estimation of the lead-times can be treated as a random variable in new policy, which can still work very well because there is no inventory drift in net stock. This advantage in new policy is illustruated by spreadsheet simulation compared to DE-APVIOBPCS model. Zero inventory drift variant works much better than the DE-

APVIOBPCS model in all facets concerned.

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## The Analysis of Zero Inventory Drift Variant Based on DE-APVIOBPCS Model

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[Abstract] The DE-APVIOBPCS model with MMSE forecast for AR(1) demand in a single-echelon supply chain has first been described in control engineering perspective. By applying the Final Value Theorem, a final value offset (i. e. inventory drift) can be measured and does exist even though the actual lead-time is known. Thus to eliminate the inherent offset and keep the system variances acceptable, a new policy with zero inventory drift based on DE-APVIOBPCS model is presented. The analysis of the variance amplification suggests the lead-times conservatively in new policy should be always estimated. The general stability conditions for zero inventory drift variant are evaluated in succession and some valuable attributes of new policy are illustrated via simulation under the assumption that misidentification of lead-time is inevitable.

[Key words] lead-time estimation inventory drift variance amplification replenishment rule z-transform

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### Research and Implementation of SYN Flood Attack Defense System

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[Abstract] There are some shortcomings of the methods commonly used in defense of SYN Flood attack. In terms of attack simulation test, I designed a defense system to SYN Flood attack. Firstly, the principle of SYN Flood attack is introduced, and then put forward the design and realization of the defense system, on this basis, according to the objectives and content of the defense attacks test, carried out the simulation attack and defense At last the implementation of the attack and defense is introduced in detail. The experimental results showed that this system can be an effective defense to SYN Flood attack, and had certain practical value.

[Key words] SYN Flood attack defence intrusion prevention