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分布时滞离散系统的全局鲁棒稳定性准则

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摘 要 研究了具有分布时滞的离散系统的全局渐进鲁棒稳定性。依据李雅普诺夫方法,和线性矩阵不等式的方法,得到了参数不确定时滞相关的全局均方渐进稳定性准则。最后,给出一个数值实例演示得到结果的有效性和可行性。

关键词 离散系统 全局渐进稳定 李雅普诺夫方法 线性矩阵不等式
中图法分类号 O231.1; **文献标志码** A

In recent years, considerable attention has been devoted to study of discrete-time system with state delay, they have strong background in engineering applications, among which network based control has been well recognized to be a typical example. Consequently much effort has been made towards investigating the stability of discrete time-delay system via Lyapunov approach[1—7]. In ref. [3], the robust stability analysis for discrete-time stochastic neural networks with time-varying delays was considered. In ref. [4], authors studied the problem of synchronization for stochastic discrete-time drive-response networks with time-varying delay. In ref. [5], they have discussed the synchronization and state estimation for discrete-time complex networks. In ref. [7], Luo have further studied the mean square exponential stability of the discrete-time stochastic neural networks with time-varying delays. To the best of our knowledge, the global asymptotic stability for discrete-time system with distributed delays has never been tackled.

In this paper, we will study the global asymptotic stability of discrete-time system with distributed delays. By utilizing a Lyapunov function and using some well-know inequalities, a unified linear matrix inequality (LMI) approach is development to establish sufficient conditions for the system to be global, asymptotically stable and obtain several less conservative conditions. A simulation example is given to show the effectiveness and less conservatism of the proposed criteria.

Notation: Throughout this letter, R^n the n dimensional Euclidean space. The superscripe "T" denotes matrix trasposition. The notation $X \geqslant Y$ (respective, $X > Y$) mean that X and Y are symmetric matrices, and that $X - Y$ is positive semi-definite (respective, positive definite). $\|\cdot\|$ is the Euclidean norm in R^n. Moreover, the asterisk $*$ in a matrix is used to denote term that is induced by symmetry matrix, if not explicitly specified, are assumed to have compatible dimensions. Sometimes, the arguments of function will be omitted in the analysis when no confusion can arise.

1 Model descriptions and preliminaries

Consider the following discrete-time system (1) with distributed delays:

$$x(k+1) = A(k)x(k) + B(k)x(k-\tau(k)) + C(k)\sum_{m=1}^{+\infty}\mu_m x(k-m) \qquad (1)$$

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Where $x(k)\in R^n$ is the state vector, $\tau(k)$ denote the time-varying delays satisfying: $\tau_m\leqslant\tau(k)\leqslant\tau_M$. The constants $\mu_m\geqslant 0$ satisfy the following convergent conditions: $\sum_{m=1}^{+\infty}\mu_m<\bar{\mu}$, $\sum_{m=1}^{+\infty}m\mu_m<+\infty$. The initial conditions of system (1) is assumed to be $x(s)=\varphi(s), s=0,-1,\cdots$. We denote: $A(k)=A+\Delta A(k), B(k)=B+\Delta B(k), C(k)=C+\Delta C(k)$. A, B, C are known constant matrices with appropriate dimensions, $\Delta A(k), \Delta B(k), \Delta C(k)$ are unknown matrices denoting the uncertainties in the system. In this paper, the uncertainties are norm-bouned and are assumed to be of the following form: $[\Delta A(k), \Delta B(k), \Delta C(k)]=DF(k)[E_1, E_2, E_3]$. Where D, E_1, E_2, E_3 are known constant matrices with appropriate dimensions, $F(k)$ are unknown time-varying functions satisfying $F^{\mathrm{T}}(k)F(k)\leqslant I$.

To obtain our results, we need introduce the following definition and lemmas.

Definition 1 The system (1) is globally asyptotically stable in the mean square if $\lim_{k\to+\infty}x^2(k,\psi)=0$.

Lemma1 Let $M\in R^{n\times n}$ be a positive semidefinite matrix, $x_i\in R^n$ and scalar constant $a_i\geqslant 0(i=1,2,\cdots)$, if the series concerned is convergent, then the following inequality holds:

$$\left(\sum_{i=1}^{+\infty}a_ix_i\right)^{\mathrm{T}}M\left(\sum_{i=1}^{+\infty}a_ix_i\right)\leqslant\left(\sum_{i=1}^{+\infty}a_i\right)\left(\sum_{i=1}^{+\infty}a_ix_i^{\mathrm{T}}Mx_i\right)$$

Lemma 2 (Schur complement) Given constant symmetric matrices $\sum_1, \sum_2, \sum_3$, where $\sum_1=\sum_1^{\mathrm{T}}$ and $0<\sum_2=\sum_2^{\mathrm{T}}$ then $\sum_1+\sum_3^{\mathrm{T}}\sum_2^{-1}\sum_3<0$ if and only if that

$$\begin{bmatrix}\sum_1 & \sum_3^{\mathrm{T}}\\ \sum_3 & -\sum_2\end{bmatrix}<0 \text{ or } \begin{bmatrix}-\sum_2 & \sum_3\\ \sum_3^{\mathrm{T}} & \sum_1\end{bmatrix}<0$$

Lemma 3 For given matrices D, E and F with $F^{\mathrm{T}}F\leqslant I$ and a scalar $\varepsilon>0$, the following inequality holds:

$$DFE+E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}}\leqslant\varepsilon DD^{\mathrm{T}}+\varepsilon^{-1}E^{\mathrm{T}}E.$$

2 Main results

Theorem 1 The system (1) is globally robust asymptotically stable if there exist ε, five positive define matrices P, Q, R, Z, U and any matrices T, M, N, such the following LMIs hold:

$$\Phi=\begin{bmatrix}\Omega+Y+Y^{\mathrm{T}}+\Sigma+\Sigma^{\mathrm{T}}+\varepsilon\alpha^{\mathrm{T}}\alpha & \sqrt{\tau_M}M & \sqrt{\tau_M-\tau_m}N & TD\\ \cdot & -Z & 0 & 0\\ \cdot & \cdot & -Z & 0\\ \cdot & \cdot & \cdot & -\varepsilon I\end{bmatrix}<0$$

Where $\Omega=(\Omega_{ij})_{7\times7}, Y=T[-(A-I), -B, 0, I, 0, 0, -C]$.

$\Sigma=[M, N-M, N, 0, 0, 0, 0], \alpha=[-E_1, E_2, 0, 0, 0, 0, E_3]$.

$\Omega_{11}=\bar{\mu}U+\beta Q_{11}+R_{11}, \Omega_{13}=P_{12}, \Omega_{14}=P_{11}+\beta Q_{12}+R_{12}$.

$\Omega_{22}=-Q_{11}, \Omega_{25}=-Q_{12}, \Omega_{33}=P_{22}-R_{11}, \Omega_{34}=P_{12}$.

$\Omega_{44}=P_{11}+\beta Q_{22}+R_{22}, \Omega_{46}=P_{12}, \Omega_{55}=-Q_{22}, \Omega_{66}=-R_{22}$,

$\Omega_{77}=-\frac{1}{\bar{\mu}}U, \beta=\tau_M-\tau_m+1$.

Proof we choose the following Lyapunov function candidate $v(k)=\sum_{i=1}^{6}v_i(k)$.

$$v_1(k)=\begin{bmatrix}x(k)\\ x(k-\tau_M)\end{bmatrix}^{\mathrm{T}}P\begin{bmatrix}x(k)\\ x(k-\tau_M)\end{bmatrix},$$

$$v_2(k)=\sum_{i=k-\tau(k)}^{k-1}\lambda^{\mathrm{T}}(i)Q\lambda(i),$$

$$v_3(k)=\sum_{j=k+1-\tau_M}^{k-\tau_m}\sum_{i=j}^{k-1}\lambda^{\mathrm{T}}(i)Q\lambda(i),$$

$$v_4(k)=\sum_{i=k-\tau_M}^{k-1}\lambda^{\mathrm{T}}(i)R\lambda(i),$$

$$v_5(k)=\sum_{i=-\tau_M}^{-1}\sum_{m=k+1}^{k-1}\lambda^{\mathrm{T}}(i)Z\lambda(i),$$

$$v_6(k)=\sum_{i=1}^{+\infty}\mu_m\sum_{j=k-i}^{k-1}x^{\mathrm{T}}(j)Ux(j).$$

Where $\lambda^{\mathrm{T}}(k)=(x^{\mathrm{T}}(k), \eta^{\mathrm{T}}(k))^{\mathrm{T}}, \eta(k)=x(k+1)-x(k)$.

Calculating the difference of $v(k)$ along the trajectories of system (1), we obtain:

$$\Delta v_1(k)=\begin{pmatrix} x(k)+\eta(k) \\ x(k-\tau_M)+\eta(k-\tau_M)\end{pmatrix}^{\mathrm{T}} P\begin{pmatrix} x(k)+\eta(k) \\ x(k-\tau_M)+\eta(k-\tau_M)\end{pmatrix} - \begin{pmatrix} x(k) \\ x(k-\tau_M)\end{pmatrix}^{\mathrm{T}} P\begin{pmatrix} x(k) \\ x(k-\tau_M)\end{pmatrix}.$$

$$\Delta v_2(k)\leqslant \lambda^{\mathrm{T}}(k)Q\lambda(k)-\lambda^{\mathrm{T}}(k-\tau(k))Q\lambda(k-\tau(k))+\sum_{i=k-\tau_M+1}^{k-\tau_m}\lambda^{\mathrm{T}}(i)Q\lambda(i).$$

$$\Delta v_3(k)=(\tau_M-\tau_m)\lambda^{\mathrm{T}}(k)Q\lambda(k)-\sum_{i=k-\tau_M+1}^{k-\tau_m}\lambda^{\mathrm{T}}(i)Q\lambda(i),$$

$$\Delta v_4(k)=\lambda^{\mathrm{T}}(k)R\lambda(k)-\lambda^{\mathrm{T}}(k-\tau(k))R\lambda(k-\tau(k)),$$

$$\Delta v_5(k)=\tau_M\eta^{\mathrm{T}}(k)Z\eta(k)-\sum_{l=k-\tau(k)}^{k-1}\eta^{\mathrm{T}}(l)Z\eta(l)-\sum_{l=k-\tau_M}^{k-\tau(k)+1}\eta^{\mathrm{T}}(l)Z\eta(l),$$

$$\Delta V_6(k)=\sum_{i=1}^{+\infty}u_i(x^{\mathrm{T}}(k)Ux(k)-x^{\mathrm{T}}(k+1)Ux(k+1))(\text{by lemma 3})\leqslant\bar{\mu}x^{\mathrm{T}}(k)Ux(k)-\frac{1}{\bar{\mu}}(\sum_{m=1}^{+\infty}u_m x(k-m))^{\mathrm{T}}U(\sum_{m=1}^{+\infty}u_m x(k-m)).$$

In addition we have $\eta(k)-(x(k+1)-x(k))=0$. So for any matrix T, we can get $\theta^{\mathrm{T}}(k)(\Theta+\Theta^{\mathrm{T}})\theta(k)=0$. where $\Theta=T(-(A(k)-I),-B(k),0,I,0,0,-C(k))$, $\theta_1^{\mathrm{T}}(k)=[x^{\mathrm{T}}(k),x^{\mathrm{T}}(k-\tau(k)),x^{\mathrm{T}}(k-\tau_M),\eta^{\mathrm{T}}(k),\eta^{\mathrm{T}}(k-\tau(k)),\eta^{\mathrm{T}}(k-\tau_M),(\sum_{m=1}^{+\infty}u_m x(k-m))^{\mathrm{T}}]$.

By lemma 2 we have

$$\theta^{\mathrm{T}}(k)(\Theta+\Theta^{\mathrm{T}})\theta(k)\leqslant\theta^{\mathrm{T}}(k)(Y+Y^{\mathrm{T}}+\varepsilon^{-1}TDD^{\mathrm{T}}T^{\mathrm{T}}+\varepsilon\alpha\alpha^{\mathrm{T}})\theta(k).$$

So we can get

$$\Delta v(k)\leqslant\theta^{\mathrm{T}}(k)(\Omega+Y+Y^{\mathrm{T}}+\varepsilon^{-1}TDD^{\mathrm{T}}T^{\mathrm{T}}+\varepsilon\alpha\alpha^{\mathrm{T}})\theta(k)-\sum_{l=k-\tau(k)}^{k-1}(\theta^{\mathrm{T}}(k)M+\eta^{\mathrm{T}}(l)Z)Z^{-1}(M^{\mathrm{T}}\times\theta(k)+Z\eta(l))-\sum_{l=k-\tau_M}^{k-\tau(k)-1}(\theta^{\mathrm{T}}(k)N+\eta^{\mathrm{T}}(l)Z)Z^{-1}\times(N^{\mathrm{T}}\theta(k)+Z\eta(l))\leqslant\theta^{\mathrm{T}}(k)\Phi\theta(k).$$

By applying lemma 1, we can get $\Delta v(k)<0$ and the asymptotic stability is established.

3 Numeral example

Consider the system (1) with the following parameters:

$$A=\begin{bmatrix}0.8 & 0\\ 0 & 0.9\end{bmatrix}, B=\begin{bmatrix}-0.1 & 0\\ -0.2 & -0.1\end{bmatrix},$$

$$C=\begin{bmatrix}-0.2 & 0.1\\ 0.2 & 0.1\end{bmatrix},$$

$\tau_M=5$, $\tau_m=2$, $D=E_1=E_2=E_3=0.1$, $\mu=1/8$.

By the Matlab Control Toolbox, we obtain:

$$P=\begin{bmatrix}20.23 & -3.16 & -4.44 & -2.01\\ -3.16 & 20.45 & -0.28 & -4.55\\ -4.44 & -0.28 & 12.0 & -0.42\\ -2.01 & -4.55 & -0.42 & 12.71\end{bmatrix},$$

$$Z=\begin{bmatrix}101.32 & -2.27\\ -2.27 & 98.41\end{bmatrix},$$

$$Q=\begin{bmatrix}1.19 & -0.16 & -0.09 & 0.8\\ -0.16 & 13.35 & 32.84 & -2.16\\ -0.09 & 32.84 & 32.84 & -2.62\\ 0.8 & -2.16 & -2.62 & 35.54\end{bmatrix},$$

$$U=\begin{bmatrix}5.81 & 0.17\\ 0.17 & 6.67\end{bmatrix},$$

$$R=\begin{bmatrix}37.52 & -0.97 & 3.03 & 0.76\\ -0.97 & 38.57 & 0.07 & 0.51\\ 3.03 & 0.07 & 37.516 & -0.97\\ 0.76 & 0.51 & -0.97 & 38.57\end{bmatrix},$$

$\varepsilon=69.57$.

4 Conclusion

In this paper, we obtained sufficient conditions for globally robust asymptotic stability of discrete-time system in terms of LMI. By introducing Lyapunov-Krasovskii functional, a new stability criterion is established. Finally, one example is given to show the superiority of our proposed stability conditions.

References

1 Liang J, Cao J, Ho D W C. Discrete time bidirectional associative memory neural networks with variable delays. Phy Letts A,2005;335(23):226—234

2 Liang J,Wang Z,Liu X. Exponential synchronization of stochastic delayed discrete-time complex networks. Nonlinear Dynamics,2008;53(1-2):153—165

3 Liu Y,Wang Z,Liu X. Robust stability of discrete-time stochastic neural networks with time-varying delays. Neuracomputing, 2008; 71: 823—833

4 Liang J,Wang Z,Liu Y,*et al*. Global synchronization control of general delayed discrete-time networks with stochastic coupling and disturbances. IEEE Transactions on Systems, Man, and Cybernetics-Part B,2008;38 (4):1073—1083

5 Liu Y,Wang Z,Liang J,*et al*. Synchronization and state estimation for discrete-time complex networks with distributed delays. IEEE Transactions on Systems, Man, and Cybernetics-Part B, 2008; 38 (5): 1314—1325

6 Wang Z,Shu H,Liu Y,*et al*. Robust stability analysis of generalized neural networks with discrete and distributed time delays. Chaos, Solution and Fractal,2006;30 (4):886—896

7 Luo Mengzhuo,Zhong Shouming,Wang Rongjun,*et al*. Robust stability analysis for discrete-time stochastic neural networks systems with time-varying delays. Appl Math Computer. 2009;209(2):305—315

Global Robust Stability Criterial for Discrete-time System with Distributed Delays

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[**Abstract**] The global asymptotic stability of discrete-time system with distributed delays is investigated. Using the Lyapunov stability theory and LMIs approaches, delay-dependent criteria are derived to ensure the global robust asymptotic stability of the addressed system in the mean square for all admissible parameter uncertainties. A numerical example is given to illustrate the effectiveness of our result.

[**Key words**] discrete-time system　the global asymptotic stability　Lyapnuov method　LMIs